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$a^2 + b^2 = c^2$

divide by a^2, b^2, a^2 –

[illegible]

ELEMENTS

OF

PLANE AND SPHERICAL TRIGONOMETRY;

WITH

PRACTICAL APPLICATIONS.

BY BENJAMIN GREENLEAF, A. M.,
AUTHOR OF A MATHEMATICAL SERIES.

IMPROVED ELECTROTYPE EDITION.

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PREFACE.

THE preparation of this treatise was undertaken at the earnest solicitation of many teachers, who, having used the author's Arithmetics, Algebras, and Elements of Geometry with satisfaction, were desirous of seeing his series rendered more nearly complete by the addition of the Elements of Plane and Spherical Trigonometry. It is hoped that the present work will be found to be a complete system, theoretical and practical, fully adapted to the wants of advanced classes.

The trigonometric functions, in this treatise, have been regarded as *ratios*, since this improved method has not only now superseded the ancient method in English and French works, but has been approved and adopted generally by the best American mathematicians. Reference, however, is made to whatever is especially valuable in the old method.

In the preparation of this work the author has received valuable suggestions from many eminent teachers, to whom he would here express his sincere thanks. Especially would he acknowledge his great obligations to H. B. Maglathlin, A. M., who for many months has been associated with him in his labors, and to whose experience as a teacher, skill as a mathematician, and ability as a writer, the value of this treatise is largely due.

BENJAMIN GREENLEAF.

BRADFORD, Mass., July 25, 1861.

NOTICE.

THIS System of Trigonometry was designed to be published in connection with the author's Elements of Geometry. It is now published *separately*, to accommodate those teachers who desire to use it in that form.

January 1, 1863.

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ELEMENTS

OF

PLANE AND SPHERICAL TRIGONOMETRY;

WITH

PRACTICAL APPLICATIONS.

TRIGONOMETRY.

BOOK I.

LOGARITHMS.

1. **The Logarithm** of a number is the exponent of the power to which a given fixed number must be raised in order to produce the first number.

2. The **Base** of the system is the fixed number.

3. The base, in the common system of logarithms, is 10. Hence, since

$10^0 = 1$, 0 is the logarithm of 1;
$10^1 = 10$, 1 " " 10;
$10^2 = 100$, 2 " " 100;
$10^3 = 1{,}000$, 3 " " 1,000;
$10^4 = 10{,}000$, 4 " " 10,000;
&c., &c.

It thus appears that, in the common system, the logarithm of every number between 1 and 10 is some number between 0 and 1; that is, a proper fraction. The logarithm of every number between 10 and 100 is some number between 1 and 2; that is, 1 plus a fraction. The logarithm of every number between 100 and 1,000 is some number between 2 and 3; that is, 2 plus a fraction; and so on.

4. By means of *negative* exponents the application of logarithms may be extended, in the common system, to numbers less than 1. Thus, since

$10^{-1} = 0.1$, -1 is the logarithm of 0.1;
$10^{-2} = 0.01$, -2 " " 0.01;
$10^{-3} = 0.001$, -3 " " 0.001;
$10^{-4} = 0.0001$, -4 " " 0.0001;
&c., &c.

From this it appears that the logarithm of every number between 1 and 0.1 is some number between 0 and —1; that is, —1 plus a fraction. The logarithm of every number between 0.1 and 0.01 is some number between —1 and —2; that is, —2 plus a fraction. The logarithm of every number between 0.01 and 0.001 is some number between —2 and —3; that is, —3 plus a fraction; and so on.

5. In the common system, as the logarithms of all numbers which are not exact powers of 10 are incommensurable with those numbers, their values can only be obtained approximately, and are expressed by decimals.

6. The *integral* part of any logarithm is called the CHARACTERISTIC, and the *decimal* part is sometimes called the MANTISSA.

7. *The characteristic of the logarithm of* ANY NUMBER GREATER THAN UNITY, *is one less than the number of integral figures in the given number.*

For it has been shown (Art. 3) that the logarithm of 1 is 0, of 10 is 1, of 100 is 2, of 1000 is 3, and so on.

8. *The characteristic of the logarithm of* ANY DECIMAL FRACTION *is a negative number, and is one more than the number of ciphers between the decimal point and the first significant figure.*

For it has been shown (Art. 4) that the logarithm of 0.1 is —1, of 0.01 is —2, of 0.001 is —3, and so on.

NOTE. — In general, whether the given number be integral, fractional, or mixed, *the characteristic of the logarithm of any number expressed decimally is the distance of the first, or left-hand, significant figure from the units' place, being positive when that figure is on the left of the units' place, and negative when on the right.*

GENERAL PROPERTIES OF LOGARITHMS.

9. *The logarithm of a* PRODUCT *is equal to the sum of the logarithms of its factors.*

For let M and N be any two numbers, x and y their respective logarithms, and a the base of the system. Then, by definition (Art. 1), we have

$$M = a^x, \quad N = a^y.$$

Multiplying equations, member by member, we have

$$MN = a^x a^y = a^{x+y}.$$

Therefore, $\log (M \times N) = x + y = \log M + \log N.$

10. *The logarithm of a* QUOTIENT *is equal to the logarithm of the dividend diminished by that of the divisor.*

For, by Art. 9, we have

$$M = a^x, \quad N = a^y.$$

Dividing the first equation by the second, member by member, we have

$$\frac{M}{N} = \frac{a^x}{a^y} = a^{x-y}.$$

Therefore, $\log \left(\frac{M}{N}\right) = x - y = \log M - \log N.$

11. *The logarithm of any* POWER *of a number is equal to the product of the logarithm of the number by the exponent of the power.*

For let m be any number, and take the equation (Art. 9)

$$M = a^x,$$

then, raising both sides to the mth power, we have

$$M^m = (a^x)^m = a^{xm}.$$

Therefore, $\log (M^m) = xm = (\log M) \times m.$

12. *The logarithm of the* ROOT *of any number is equal to the logarithm of the number divided by the index of the root.*

For, let n be any number, and take the equation (Art. 9)

$$M = a^x,$$

then, extracting the nth root of both sides, we have

$$\sqrt[n]{M} = \sqrt[n]{a^x} = a^{\frac{x}{n}}.$$

Therefore, $$\log(\sqrt[n]{M}) = \frac{x}{n} = \frac{\log M}{n}.$$

13. Hence, by means of logarithms, we can perform multiplication by addition, and division by subtraction; also, we can raise a number to any power by a single multiplication, and extract any root of a number by a single division.

14. *All numbers, integral, fractional, or mixed, having the same succession of significant figures, have logarithms with the same decimal part.*

For since the logarithm of 10 is 1, the product of any number by 10 will have a logarithm increased by 1; and, likewise, the quotient of any number divided by 10 will have a logarithm diminished by 1; and, 1 being an integer, the logarithms will differ only in their characteristics.

Thus, the logarithm	of	235	is	2.371068
"	"	2350	"	3.371068
"	"	23.5	"	1.371068
"	"	2.35	"	0.371068
"	"	.235	"	$\bar{1}$.371068
"	"	.0235	"	$\bar{2}$.371068

15. The negative sign placed over the characteristic indicates that it alone is negative, the decimal part being always positive.

TABLE OF LOGARITHMS.

16. A Table of Logarithms usually contains all the whole numbers between 1 and a given number, with their logarithms. The accompanying table contains the logarithms of all numbers from 1 up to 10,000, calculated to six places of decimals.

17. In the table, the characteristics of the logarithms of the first 100 numbers are inserted; but for all other numbers the decimal part only of the logarithms is given, while the characteristic is left to be supplied by inspection, according to the principles already furnished (Art. 7, 8).

18. The numbers are in the column headed N, and their logarithms, or the decimal parts of their logarithms, are opposite

on the same line. When the first two figures of the decimal are the same for several successive logarithms, they are not repeated for each, but, being used once, are then left to be supplied.

19. In the column headed D are the mean or average differences of the ten logarithms against which they are placed.

TO FIND THE LOGARITHM OF ANY NUMBER.

20. *When the given number is any integer of* ONE *or* TWO *figures.*

Look on the first page of the table, and opposite the given number will be found the logarithm with its characteristic. Thus,

the logarithm of 63 is 1.799341;
" " 98 " 1.991226.

21. *When the given number is any integer of* THREE FIGURES.

Look in the table for the given number, and opposite the same, in the column headed 0, will be found the decimal part of the logarithm, to which must be prefixed 2 as the characteristic (Art. 7). Thus,

the logarithm of 110 is 2.041393;
" " 817 " 2.912222.

22. *When the given number is any integer of* FOUR *figures, either with or without ciphers annexed.*

Find the first three figures of the given number in the column headed N, and, opposite to them, in the column headed by the fourth figure, will be found the decimal part of the logarithm; to which the characteristic, as determined by Art. 7, must be prefixed. Thus,

the logarithm of 4901 is 3.690285;
" " 9677 " 3.985741;
" " 31250 " 4.494850.

23. *When the given number is any integer of* FIVE *or* MORE *figures.*

Find the logarithm of the first four figures as in Art. 22, regarding the others as ciphers annexed; then take, from the column headed D, the number on the same horizontal line with the decimal part of the logarithm, and multiply it by the remaining figures of the given number; reject from the right of the product thus obtained as many figures as there were in the multiplier, and add what is left to the decimal part of the logarithm already found; and the sum will be the required logarithm. Thus, if it be required to find the logarithm of 93192:

the logarithm of 93190 is	4.969369	Dif. from col. D,	47
	9.4		2
" " 93192 "	4.969378.		9.4

This process is based upon the supposition that the differences of logarithms are proportional to the differences of their corresponding numbers, which is not strictly correct, yet sufficiently exact for practical purposes.

When the figure or figures rejected from the right of the product, considered decimally, exceed in value .5, the right-hand figure of what is left to be added must be increased by 1, to insure greater accuracy in the result.

24. *When the given number is any* DECIMAL FRACTION, *or any mixed number expressed decimally.*

Find the decimal part of the logarithm of the number, as if it were an integer, and prefix the proper characteristic (Arts. 7 and 8). Thus,

the logarithm of 93.192 is 1.969378;
" " 4526.375 " 3.655751;
" " .0006801 " $\bar{4}.832573$.

25. *When the given number is any* COMMON FRACTION.

Reduce the given fraction to a decimal, and find its logarithm, as in Art. 24; or, since a fraction is an expression of division, subtract the logarithm of the denominator from the logarithm of the numerator, and the difference will be the logarithm of the fraction (Art. 10). Thus,

the logarithm of $\frac{3}{4}$, or .75, is $\bar{1}.875061$.

Or,

the logarithm of the numerator, 3, is 0.477121
" " " denominator, 4, " 0.602060
" " " fraction, $\frac{3}{4}$ " $\bar{1}.875061$

To find the Number corresponding to any Logarithm.

26. *When the given logarithm is* WITHIN *the limits of the table.*

Look in the column headed 0, for the first two or three figures of the logarithm, neglecting the characteristic, and, if *all* the figures of the logarithm are found in that column, the corresponding number will be on the same horizontal line, in the column headed N. If, however, the decimal part of the logarithm be not exactly found in the column headed 0, look for it in one of the nine following columns, and the first three figures of the corresponding number will be on the same horizontal line in the column headed N, and the fourth will be at the head of the column in which the logarithm was found.

Make the number correspond with the characteristic given, if necessary, by pointing off decimals or annexing ciphers (Arts. 7 and 8). Thus,

the number corresponding to the logarithm 3.146128 is 1400;
" " " 0.370143 " 2.345;
" " " $\bar{2}.907680$ " .08085.

27. *When the given logarithm is* NOT WITHIN *the limits of the table.*

From the decimal part of the given logarithm subtract the decimal part of that next less; annex to their difference two or more ciphers, and divide by the number, in the column headed D, opposite the decimal part taken from the table. Annex the result to the number corresponding to the lesser logarithm, and point off according to the characteristic, as before.

It sometimes happens, in dividing by the tabular difference, that there are not as many figures in the quotient as there are ciphers annexed to the dividend. In such a case, supply the deficiency, as in the division of decimals, by prefixing a cipher or ciphers to the quotient before annexing.

This process, like its converse (Art. 23), is based upon the supposition that the differences of logarithms are proportional to the differences of their corresponding numbers.

NOTE. The number corresponding to a given logarithm, when obtained by the use of a table calculated to six decimal places, is reliable only to the sixth figure, and sometimes that figure of an answer is not strictly correct.

Let it be required to find the number corresponding to the logarithm 2.633356.

The dec. part of the given log. is .633356
" " log. next less is .633266, correspon. num., 4298
Their difference is / Difference from column D is $\frac{90.00}{101} =$ 89
Logarithm 2.633356 has for its corresponding number 429.889

The number corresponding to the logarithm 3.441049 is 2760.89
" " " " $\bar{2}$.497935 is .0314728
" " " " 2.436811 is 273.408

ARITHMETICAL COMPLEMENT.

28. The arithmetical complement of any logarithm is the difference between it and 10.

Thus, the arithmetical complement of the logarithm of 41, is $10 - \log 41 = 10 - 1.612784 = 8.387216$.

29. The arithmetical complement of a logarithm may be readily found, from the table, *by subtracting each figure of the logarithm from* 9, *excepting the last significant figure at the right, which must be taken from* 10; for this is equivalent to subtracting the logarithm from 10.

30. *The difference of two logarithms is equal to the sum of the logarithm to be diminished and the arithmetical complement of the other, less* 10.

For let a be any logarithm, and b a logarithm to be subtracted from it; then their difference will be $a - b$.

Now the arithmetical complement of b is $10 - b$; adding $10 - b$ to a, we have $a + 10 - b$; subtracting 10, we have

$$a+10-b-10=a-b;$$

the same result as before.

31. Hence, since an arithmetical complement added makes the result 10 too great, a corresponding allowance must be made in any operation in which arithmetical complements of logarithms are used.

MULTIPLICATION BY LOGARITHMS.

32. *Add the logarithms of the factors; and the sum will be the logarithm of their product* (Art. 9).

The term *sum*, here used, is to be understood in its *algebraic* signification. Therefore, since the characteristic alone of a logarithm is negative (Art. 15), whatever there is to be carried from the decimal part, in the operation, must either be added to a positive characteristic, or subtracted from one that is negative. Also, when the characteristics of the logarithms are not either all positive or all negative, the difference between their sums must be taken, and the sign of the greater prefixed.

Ex. 1. Multiply 120, 101, and .015 together.

$$\begin{array}{lcr} \text{Log } 120 & = & 2.079181 \\ \text{“ } 101 & = & 2.004321 \\ \text{“ } .015 & = & \underline{\bar{2}.176091} \\ \text{Product} = 181.8 \ldots\ldots & & 2.259593 \end{array}$$

Here, the $\bar{2}$ cancels a positive 2, so we have but 2 to set down.

2. Multiply 3.26 by .0085. Ans. .02771.

3. Multiply 6651 by 108. Ans. 718308.

DIVISION BY LOGARITHMS.

33. *Subtract the logarithm of the divisor from the logarithm of the dividend, and the difference will be the logarithm of their quotient* (Art. 10). Or,

Add the arithmetical complement of the logarithm of the divisor to the logarithm of the dividend, and the sum, less 10, *will be the logarithm of the quotient* (Art. 31).

The term *difference*, here used, is to be understood in its *algebraic* signification. Therefore, the sign of the characteristic of the divisor must be changed; and then, if the characteristics of the divisor and dividend have the same sign, their sum must be taken, but when of different signs, their difference, with the sign of the greater, for the characteristic of the logarithm of the quotient. Also, if 1 is carried from the decimal part, it must be regarded as positive, and must be united with the characteristic of the divisor before it is changed.

Ex. 1. Divide 850 by .093.

FIRST OPERATION.			SECOND OPERATION.		
Log 850	=	2.929419	Log 850	=	2.929419
" .093	=	$\bar{2}.968483$	" .093 ar. co.	=	11.031517
Quot. 9139.8	.....	3.960936	Quot. 9139.8	.....	3.960936

Here, in the first operation, 1 carried from the decimal part to the $\bar{2}$ changes it to $\bar{1}$, which being taken from 2, leaves 3 to set down; and, in the second operation, 10 is taken from the sum of the characteristics (Art. 31).

2. Divide 2625 by 125. Ans. 21.
3. Divide .02771 by .0085. Ans. 3.26.
4. Divide 117.1347 by 5.062. Ans. 23.14.
5. Find the 4th term of the proportion,

219 : 63.05 :: 378.

Log	378	=	2.577492
"	63.05	=	1.799685
"	219 ar. co.	=	7.659556
4th term = 108.826			2.036733

6. Find the 4th term of the proportion, 720 : 196 :: 155.5. Ans. 42.33.

INVOLUTION BY LOGARITHMS.

34. *Multiply the logarithm of the given number by the exponent of the power to which the number is to be raised; and the product will be the logarithm of the required power* (Art. 11).

Since the exponent of any power is positive, a negative characteristic multiplied by it will give a negative result; but that which is to be carried from the decimal part will be positive; therefore, their difference will be the characteristic of the product.

Ex. 1. Required the square, or second power, of 31.

$$\begin{array}{lcr} \text{Log } 31 & = & 1.491362 \\ & & \underline{2} \\ \text{Ans. } 961 & & 2.982724 \end{array}$$

2. Required the cube, or third power, of .25.

$$\begin{array}{lcr} \text{Log } .25 & = & \bar{1}.397940 \\ & & \underline{3} \\ \text{Ans. } 0.015625 & & \bar{2}.193820 \end{array}$$

3. Required the tenth power of .64. Ans. .0115292.

Evolution by Logarithms.

35. *Divide the logarithm of the given number by the index of the root; and the quotient will be the logarithm of the required root* (Art. 12).

When the characteristic of the logarithm is negative, and does not contain the given divisor without a remainder, we may increase the characteristic by any number that will make it exactly divisible, provided we prefix an equal positive number to the decimal part of the logarithm.

Ex. 1. Required the square, or second root, of 1296.

$$\begin{array}{lcrl} \text{Log } 1296 & = & 3.112605 & \\ (\text{Log } 1296) \div 2 & = & 1.556303 & \text{Ans. } 36. \end{array}$$

2. Required the cube, or third root, of .00048.

$$\begin{array}{lcrl} \text{Log } .00048 & = & \bar{4}.681241 & \\ (\text{Log } .00048) \div 3 & = & \bar{2}.893747 & \text{Ans. } .078297. \end{array}$$

Here, the negative characteristic $\bar{4}$ not being exactly divisible by 3, it is increased by 2 to make it so, and then the 2 borrowed is restored, by regarding 2 as prefixed to the decimal part.

3. Required the fourth root of .434296. Ans. .811794.

4. What is the tenth root of 2? Ans. 1.0718.

BOOK II.

PLANE TRIGONOMETRY.

DEFINITIONS AND ELEMENTARY PRINCIPLES.

36. TRIGONOMETRY is the science which treats of methods of computing angles and triangles.

37. PLANE TRIGONOMETRY treats of methods of computing plane angles and triangles.

38. The MAGNITUDE OF ANGLES is represented by numbers expressing how many times they contain a certain angle fixed upon as the *unit* of angular measure.

For this purpose a right angle is generally divided into 90 equal parts called *degrees*, each degree into 60 equal parts called *minutes*, each minute into 60 equal parts called *seconds*; then an angle is expressed by the number of degrees, minutes, seconds, and decimal parts of a second, which it contains.

39. Degrees, minutes, and seconds, are marked by the symbols $^\circ$, $'$, $''$; thus, to represent 16 degrees, 9 minutes, 23.5 seconds, we write $16^\circ\ 9'\ 23''.5$.

40. Since angles at the centre of a circle are to each other as the arcs of the circumference intercepted between their sides (Geom., Prop. XVII. Bk. III.), these arcs may be regarded as the measures of the angles, and the number of *units of arc* intercepted on the circumference may be used to express both the arc and the corresponding angle.

41. A *degree of arc* is $\frac{1}{360}$ of a circumference; a *minute*, $\frac{1}{60}$ of a degree; a *second*, $\frac{1}{60}$ of a minute; and these arcs subtend angles of a degree, a minute, and a second, respectively, at the centre.

42. For simplifying calculations, the radius employed in measuring angles, being constant, is taken at an assumed value of unity, as the *linear unit of measure.*

43. Since the value of the constant ratio of the circumference to the diameter of a circle, represented by π, is 3.14159 (Geom., Prop. XV. Sch. 1, Bk. VI.), if the radius of a circle is denoted by r, its circumference is $2\pi r$, where $\pi = 3.14159$. Hence, as r is taken as unity, any number of degrees may be expressed as a multiple or fractional part of π. Thus $360° = 2\pi$, $180° = \pi$, $90° = \frac{\pi}{2}$, and $30° = \frac{\pi}{6}$.

44. The COMPLEMENT OF AN ANGLE, or arc, is the remainder obtained by subtracting the angle or arc from 90°. Thus the complement of 45° is 45°, and the complement of 31° is 59°.

When an angle, or arc, is greater than 90°, its complement is negative. Thus the complement of 127° is — 37°.

Since the two acute angles of a right-angled triangle are together equal to a right angle, they are complements one of the other.

45. The SUPPLEMENT OF AN ANGLE, or arc, is the remainder obtained by subtracting the angle or arc from 180°. Thus the supplement of 110° is 70°.

When the angle is greater than 180°, its supplement is negative. Thus the supplement of 200° is — 20°.

Since the three angles of any triangle are together equal to two right angles, any one of them is a supplement of the sum of the other two.

TRIGONOMETRIC FUNCTIONS.

46. TRIGONOMETRIC FUNCTIONS are the quantities by which angles are subjected to computation.

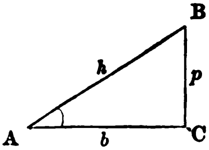
These we shall consider, in accordance with the best modern authorities, as *ratios* formed by comparing the sides of a right-angled triangle, and thus capable of comparison one with another by means of their geometrical properties.

These ratios have received the special names of *sine, tangent, secant, cosine, cotangent, and cosecant.*

There are also sometimes employed the quantities termed *versed sine, coversed sine,* and *suversed sine.*

47. *The* SINE *of an angle is the ratio of the opposite side to the hypothenuse.*

Thus, in any right-angled triangle, $A\ B\ C$, if the sides be denoted by p, b, h, we shall have,



$$\sin A = \frac{p}{h}, \quad \sin B = \frac{b}{h}. \qquad (1)$$

48. *The* TANGENT *of an angle is the ratio of the opposite side to the adjacent side.*

$$\text{Thus,} \qquad \tan A = \frac{p}{b}, \quad \tan B = \frac{b}{p}. \qquad (2)$$

49. *The* SECANT *of an angle is the ratio of the hypothenuse to the adjacent side.*

$$\text{Thus,} \qquad \sec A = \frac{h}{b}, \quad \sec B = \frac{h}{p}. \qquad (3)$$

x

prove 50. *The* COSINE, COTANGENT, *and* COSECANT *of an angle are respectively the* SINE, TANGENT, *and* SECANT *of its complement.*

Hence, since the acute angles of a right-angled triangle are complements one of the other (Art. 44), we have, according to the definitions,

$$\left.\begin{aligned} \cos A &= \sin B = \frac{b}{h}, & \cos B &= \sin A = \frac{p}{h}; \\ \cot A &= \tan B = \frac{b}{p}, & \cot B &= \tan A = \frac{p}{b}; \\ \operatorname{cosec} A &= \sec B = \frac{h}{p}, & \operatorname{cosec} B &= \sec A = \frac{h}{b}. \end{aligned}\right\} \quad (4)$$

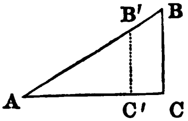
51. Since $\frac{h}{p}$ is the reciprocal of $\frac{p}{h}$, $\frac{b}{p}$ of $\frac{p}{b}$, and $\frac{b}{h}$ of $\frac{h}{b}$, we see that *the cosecant, cotangent, and cosine of an angle are respectively the reciprocals of the sine, tangent, and secant of the angle.* That is,

$$\left.\begin{aligned} &\cos A = \frac{1}{\sec A}, \quad \cot A = \frac{1}{\tan A}, \quad \operatorname{cosec} A = \frac{1}{\sin A}; \\ &\sin A = \frac{1}{\operatorname{cosec} A}, \quad \tan A = \frac{1}{\cot A}, \quad \sec A = \frac{1}{\cos A}; \\ &\sin A \operatorname{cosec} A = 1, \quad \cos A \sec A = 1, \quad \tan A \cot A = 1. \end{aligned}\right\} \quad (5)$$

52. If the cosine of A be subtracted from unity, the remainder is called the *versed sine* of A; if the sine of A be subtracted from unity, the remainder is called the *coversed sine* of A; and if the cosine of A be added to unity, the sum is called the *suversed sine* of A. Hence,

$$\left.\begin{array}{ll}\text{vers}\,A = 1 - \cos A, & \text{covers}\,A = 1 - \sin A, \\ & \text{suvers}\,A = 1 + \cos A.\end{array}\right\} \quad (6)$$

53. *The values of trigonometric ratios remain the same so long as the angle continues the same.*

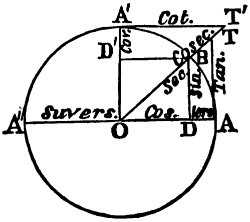


Let BAC be any angle; in AB take any point, B, and draw BC perpendicular to AC; also take any other point, B', and draw $B'C'$ perpendicular to AC. Then, since the triangles ABC, $AB'C'$ are similar, their sides have to one another the same ratio (Geom., Art. 210), and therefore $\sin A$, $\tan A$, &c. will have the same values, whether ABC or $AB'C'$ be the triangle by the sides of which they are expressed. It is also evident that their values would change with a change of the angle. Hence,

The trigonometric ratios determine the angles, and conversely; that is, any determinate values being given for the one, determinate values can be found for the other.

54. The terms sine, tangent, secant, &c., were formerly* considered to be *functions of an arc*, and denoted certain *trigonometric lines.*

Thus, let O be the centre of any circle, AA'' its diameter, and AB any arc; draw the radius OA' at right angles to AA'', and draw tangents to the circle at the points A and A'; produce OB to meet the first tangent in T and the second tangent in T'; draw BD perpendicular to OA, and BD' perpendicular to OA'. Then, by the old definitions, the *lines* of the figure are considered to



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* "The modern method has now completely superseded the ancient method in English works." — *Todhunter's Trigonometry*, p. 49.

be the functions of the arc AB. BD is the *sine* of the arc AB, OD its *cosine*, AT its *tangent*, $A'T'$ its *cotangent*, OT its *secant*, OT' its *cosecant*, AD its *versed sine*, $A'D'$ its *coversed sine*, and $A''D$ its *suversed sine*. Also the line joining A and B is the *chord* of the arc AB.

That is, in the circle whose radius is unity; —

The SINE *of an arc, or of the angle measured by that arc, is the perpendicular let fall from one extremity of the arc, upon the diameter passing through the other extremity.*

The COSINE *is the distance from the centre to the foot of the sine.*

The trigonometric TANGENT *is that part of the tangent touching one extremity of the arc, which is intercepted between that extremity and the radius produced passing through the other extremity.*

The SECANT *is that part of the radius produced which is intercepted between the centre and the tangent.*

The VERSED SINE *is that part of the diameter intercepted between the foot of the sine and the origin of the arc.*

The COTANGENT, COSECANT, *and* COVERSED SINE *are tangent, secant, and versed sine, respectively, of the complement of an arc or angle.* The cosine is also equal to the sine of the complement, as $OD = D'B$.

The SUVERSED SINE *is that part of the diameter which remains after taking away the versed sine*, or it is the versed sine of the supplement.

omit

55. If the radius of the circle be *unity*, the numerical value of the sine and other trigonometric functions is the same in both the old and new systems, for

$$\sin AOB = \frac{BD}{OB}, \qquad \sin AB = BD.$$

But OB is the radius of the circle, and denoting it by r, we have

$$\sin AB = \sin AOB \times r, \qquad \sin AOB = \frac{\sin AB}{r};$$

and making radius $= 1$, we have

$$\sin AB = \sin AOB. \tag{7}$$

In like manner it may be shown, that similar results hold for all the other trigonometric functions. Hence any formula expressed in the old system may be immediately converted into a formula expressed in the new system, by supposing the radius of the circle to be equal to unity.

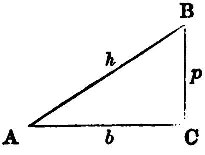
56. The sine, cosine, tangent, and cotangent constitute the *primary class* of trigonometric ratios, as they are by far most frequently used; and the others form a subordinate class, the employment of which is occasionally attended with convenience. They are collected, for more ready reference, in the following

TABLE.

1.	$\sin A = \frac{p}{h}$	1.	$\sin A = \frac{1}{\operatorname{cosec} A}$
2.	$\cos A = \frac{b}{h}$	2.	$\cos A = \frac{1}{\sec A}$
3.	$\tan A = \frac{p}{b}$	3.	$\tan A = \frac{1}{\cot A}$
4.	$\cot A = \frac{b}{p}$	4.	$\cot A = \frac{1}{\tan A}$
5.	$\sec A = \frac{h}{b}$	5.	$\sec A = \frac{1}{\cos A}$
6.	$\operatorname{cosec} A = \frac{h}{p}$	6.	$\operatorname{cosec} A = \frac{1}{\sin A}$
7.	$\operatorname{vers} A = 1 - \frac{b}{h}$	7.	$\operatorname{vers} A = 1 - \cos A$
8.	$\operatorname{covers} A = 1 - \frac{p}{h}$	8.	$\operatorname{covers} A = 1 - \sin A$
9.	$\operatorname{suvers} A = 1 + \frac{b}{h}$	9.	$\operatorname{suvers} A = 1 + \cos A$

OTHER RELATIONS BETWEEN TRIGONOMETRIC FUNCTIONS OF THE SAME ANGLE.

57. *To find the* COSINE *of an angle by means of its sine.*



From the right-angled triangle ABC (Geom., Prop. XI. Bk. IV.) we have

$$p^2 + b^2 = h^2.$$

Dividing both sides of the equation by h^2

gives $$\frac{p^2}{h^2}+\frac{b^2}{h^2}=1;$$

or, by definition (Art. 47, 50),

$$\sin^2 A+\cos^2 A=1; \tag{8}$$

therefore $$\cos^2 A=1-\sin^2 A, \tag{9}$$

and $$\cos A=\sqrt{1-\sin^2 A}; \tag{10}$$

in which "$\sin^2 A$" denotes "the square of the sine of A."

58. *To find the* SINE *of an angle by means of its cosine.*

Since, by (8), $\sin^2 A+\cos^2 A=1$,

$$\sin^2 A=1-\cos^2 A, \tag{11}$$

and $$\sin A=\sqrt{1-\cos^2 A}. \tag{12}$$

59. *To find the* TANGENT *and* COTANGENT *of an angle by means of the sine and cosine.*

By (2) we have

$$\tan A=\frac{p}{b}, \qquad \text{and} \qquad \frac{\sin A}{\cos A}=\frac{p}{h}\div\frac{b}{h}=\frac{p}{b};$$

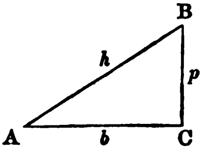
therefore $$\tan A=\frac{\sin A}{\cos A}. \tag{13}$$

Then, by Art. 51, $$\operatorname{cotan} A=\frac{\cos A}{\sin A}. \tag{14}$$

60. *To find the* SECANT *and* COSECANT *of an angle by means of the tangent.*

From the right-angled triangle ABC we have

$$h^2=b^2+p^2.$$



Dividing both sides of the equation by b^2, gives

$$\frac{h^2}{b^2}=1+\frac{p^2}{b^2};$$

or, by definitions (Art. 48, 49),

$$\sec^2 A=1+\tan^2 A. \tag{15}$$

Again, since $h^2 = p^2 + b^2$,

$$\frac{h^2}{p^2} = 1 + \frac{b^2}{p^2};$$

or, by definitions (Art. 50),

$$\operatorname{cosec}^2 A = 1 + \cot^2 A. \tag{16}$$

61. In general, any one of the six trigonometric ratios of an angle being given, the relations expressed by the foregoing formulæ will enable us to find the value of all the rest. These are termed the *elementary formulæ*, and are collected in the following

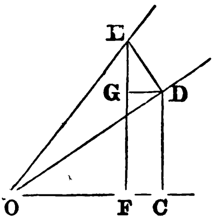
TABLE.

1. $\sin^2 A + \cos^2 A = 1.$	
2. $\sin^2 A = 1 - \cos^2 A.$	3. $\cos^2 A = 1 - \sin^2 A.$
4. $\tan A = \dfrac{\sin A}{\cos A}.$	5. $\cot A = \dfrac{\cos A}{\sin A}.$
6. $\sec^2 A = 1 + \tan^2 A.$	7. $\operatorname{cosec}^2 A = 1 + \cot^2 A.$

RELATIONS BETWEEN TRIGONOMETRIC FUNCTIONS OF DIFFERENT ANGLES.

62. *To find the* SINE *and* COSINE *of the* SUM *of two angles by means of their sines and cosines.*

Let the two angles be COD, DOE. In the line OE take any point, E, draw EF perpendicular to OC, and ED perpendicular to OD. Draw DG perpendicular to EF, and DC perpendicular to OC. The triangles GED and COD have their sides perpendicular, hence they are similar (Geom., Prop. XXV. Bk. IV.), and the angles DEG and COD are equal.



Let $a = COD = DEG$, and $b = DOE$.

Then $a + b = COE$,

and $$\sin(a+b) = \frac{EF}{OE} = \frac{GF + EG}{OE} = \frac{DC}{OE} + \frac{EG}{OE};$$

or, substituting for $\frac{DC}{OE}$ the ratios of which it is formed,

$$\frac{DC}{OD} \times \frac{OD}{OE} = \sin a \cos b,$$

and in like manner, for $\frac{EG}{OE}$,

$$\frac{EG}{ED} \times \frac{ED}{OE} = \cos a \sin b,$$

we have $$\sin(a + b) = \sin a \cos b + \cos a \sin b. \qquad (17)$$

Again, $\cos(a + b) = \frac{OF}{OE} = \frac{OC - CF}{OE} = \frac{OC}{OE} - \frac{DG}{OE}$;

or, substituting for $\frac{OC}{OE}$ the ratios of which it is formed,

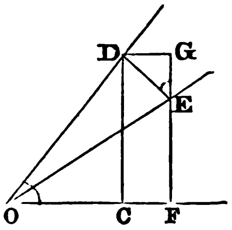
$$\frac{OC}{OD} \times \frac{OD}{OE} = \cos a \cos b,$$

and in like manner, for $\frac{DG}{OE}$,

$$\frac{DG}{DE} \times \frac{DE}{OE} = \sin a \sin b,$$

we have $$\cos(a + b) = \cos a \cos b - \sin a \sin b. \qquad (18)$$

63. *To find the* SINE *and* COSINE *of the* DIFFERENCE *of two angles by means of their sines and cosines.*



Let the two angles be FOD, DOE. In the line OE take any point, E, draw EF perpendicular to OF, and ED perpendicular to OD. Draw DG perpendicular to FE produced, and DC perpendicular to OF. The triangles GED and COD have their sides perpendicular, hence they are similar (Geom., Prop. XXV. Bk. IV.), and the angles DEG and COD are equal.

Let $a = COD = DEG$, and $b = DOE$.

Then $$a - b = FOE,$$

and $$\sin(a-b)=\frac{EF}{OE}=\frac{GF-GE}{OE}=\frac{DC}{OE}-\frac{GE}{OE};$$

or, substituting for $\frac{DC}{OE}$ the ratios of which it is formed,

$$\frac{DC}{OD}\times\frac{OD}{OE}=\sin a\cos b,$$

and in like manner, for $\frac{GE}{OE}$,

$$\frac{GE}{ED}\times\frac{ED}{OE}=\cos a\sin b,$$

we have $$\sin(a-b)=\sin a\cos b-\cos a\sin b. \qquad (19)$$

Again, $$\cos(a-b)=\frac{OF}{OE}=\frac{OC+CF}{OE}=\frac{OC}{OE}+\frac{DG}{OE};$$

or, substituting for $\frac{OC}{OE}$ the ratios of which it is formed,

$$\frac{OC}{OD}\times\frac{OD}{OE}=\cos a\cos b,$$

and in like manner, for $\frac{DG}{OE}$,

$$\frac{DG}{ED}\times\frac{ED}{OE}=\sin a\sin b,$$

we have $$\cos(a-b)=\cos a\cos b+\sin a\sin b. \qquad (20)$$

64. The four formulæ last established apply to arcs as well as angles, and may be considered the *fundamental formulæ* of subsequent analysis. They are brought together in the following

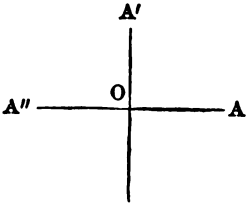
TABLE.

1. $\sin(a+b)=\sin a\cos b+\cos a\sin b.$
2. $\cos(a+b)=\cos a\cos b-\sin a\sin b.$
3. $\sin(a-b)=\sin a\cos b-\cos a\sin b.$
4. $\cos(a-b)=\cos a\cos b+\sin a\sin b.$

Read [illegible] 3.

SIGNS OF THE TRIGONOMETRIC FUNCTIONS.

65. If on any line, straight or curved, different distances be measured from a fixed point of origin, the distances which have contrary situations may by convention be introduced into our calculations, by affecting the quantities representing their magnitudes by contrary signs.



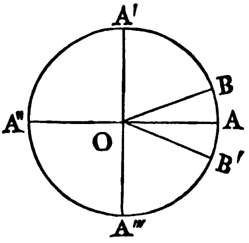
Let O be a fixed point in any line, $A A''$, and suppose we have to determine the situations of other points in this line with respect to O. The position of any point in the line will be known if we know the distance of the point from O, and also know on which side of O the point lies. Now it is found convenient to adopt the following *convention:* distances measured in one direction from O along the line will be denoted by *positive* quantities, and distances measured in the contrary direction from O will be denoted by *negative* quantities.

Thus, for example, suppose that distances measured from O towards the *right* are denoted by *positive* numbers, and let A be a point, the distance of which from O is denoted by 2 or $+2$; then if A'' be a point situated just as far to the *left* of O as A is to the *right*, the distance of A'' from O will be denoted by -2.

In like manner, if distances originating in $A A''$, and taken along $O A'$, or only parallel to $O A'$, when measured *upwards* be denoted by *positive* quantities, on being measured *downwards* they will be denoted by *negative* quantities.

66. A similar convention may conveniently be adopted with respect to angles.

Let any line, $O B$, revolve from the position $O A$, in one direction round O, forming the angle $B O A$, and let this angle be denoted by a *positive* quantity; then, if the line $O B$ revolve,



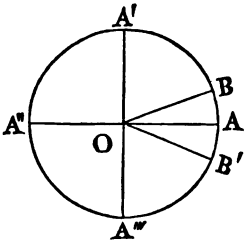
from the position OA, round O in the contrary direction, forming the angle $B'OA$, this angle may be denoted by a *negative* quantity.

Thus, for example, if each of the angles AOB and AOB' is two ninths of a right angle, and we denote the former by 20° or $+20°$, the latter may be denoted by $-20°$.

The direction of the positive distances is quite indifferent; but, being once fixed, the negative distances must lie in the contrary direction.

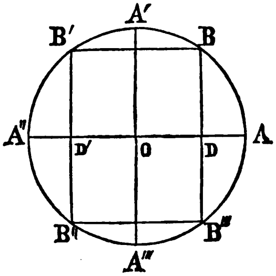
67. The representation of angles as the measure of *the revolution of a line,* turning in a plane about one of its own points, leads to the consideration of angles, not only greater than two right angles, but of all possible magnitudes.

Thus, when the line OB, starting from the initial position OA, has passed A'', or made more than half a revolution, we have described an angular magnitude of more than 180°; and when it has passed on to A, we have an angular magnitude of 360°. If it now continues to revolve in the same direction till it arrives again at B, we have an angular magnitude of $360° + 20° = 380°$, and thus we may conceive of angles of all magnitudes. In like manner negative angles of all magnitudes may be formed by the describing line OB revolving from OA, but in a contrary direction.



68. The algebraic signs of the trigonometric functions can be readily fixed in the mind by being represented geometrically. Thus,

Let the extremity of a revolving line, starting from the initial position OA, describe the positive arc AB less than 90°, AB' between 90° and 180°, $AA'B''$ between 180° and 270°, and $AA'A''B'''$ between 270° and 360°. Then, according to the definitions of Art. 54, BD, $B'D'$,



$B'D'$, and $B'''D$ are the sines, and OD and OD' are the cosines, of the angles measured by the arcs terminating in each of the four quadrants.

As all the functions of an angle less than 90° are considered positive, their direction will fix the signs for other quadrants.

It will be seen (Art. 65) that the sines are *above* the diameter AA'', and *positive*, in the first and second quadrants, but *below*, and *negative*, in the third and fourth quadrants.

Also (Art. 65), the cosines are to the *right* of the diameter $A'A'''$, and *positive*, in the first and fourth quadrants, but to the *left*, and *negative*, in the second and third quadrants.

As $\tan A = \frac{\sin A}{\cos A}$ (13), the tangent must be *positive* when the sine and cosine have the *same sign*, and *negative* when they have *unlike signs*. Hence the tangents are *positive* in the first and third quadrants, and *negative* in the second and fourth quadrants, a result which may also be obtained by noticing whether the tangents must run *above* or *below* the origin A, to meet the secant.

The *cotangent* of any angle or arc always has the same algebraic sign as its *tangent*, the *secant* the same as the *cosine*, and the *cosecant* the same as the *sine*; for they are reciprocals (Art. 51).

The versed sine, coversed sine, and suversed sine, since they are referred to the origins of their arcs, A, A', and A'', as fixed points, instead of the centre O, can have but one direction, and therefore are always *positive*.

By comparing the sine $B'''D$ and the cosine OD of the negative arc AB''' (Art. 66) with those of the equal positive arc AB, it will be seen that the cosines are identical, and consequently the secants are equal; but the sines, and, consequently, the tangents, cotangents, and cosecants, have *unlike* algebraic signs. The functions of the arc AB''', terminating in the first negative quadrant, are the same as those of the arc $AA'A''B'''$, terminating in the fourth positive quadrant. The second negative and third positive, the third negative and second positive, and the fourth negative and first positive quadrants likewise have functions with the *same* algebraic signs.

69. From the results above obtained is formed the following

TABLE.

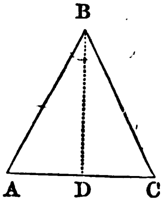
Positive Quadrant.	Sine.	Cosine.	Tangent.	Cotan't	Secant.	Cosecant.	Negative Quadrant.
First.	+	+	+	+	+	+	Fourth.
Second.	+	—	—	—	—	+	Third.
Third.	—	—	+	+	—	—	Second.
Fourth.	—	+	—	—	+	—	First.

VALUES OF THE TRIGONOMETRIC FUNCTIONS OF CERTAIN ANGLES.

70. The definitions of the trigonometric functions already given (Art. 46–50) apply directly only to angles not exceeding a right angle. But by means of the formulæ which have been deduced from them we may now extend the definitions so as to render them applicable to angles of any magnitude.

71. *To find the* SINE, *&c. of* 30° and of 60°.

Let ABC be an equilateral triangle, then each of its angles equals one third of two right angles, or 60°. Draw BD perpendicular to AC, then the angle ABD is equal to $\frac{1}{2}ABC$, AD is equal to DC, and $AD = \frac{1}{2}BC = \frac{1}{2}AB$.



Therefore,

$$\sin ABD = \frac{AD}{AB} = \frac{\frac{1}{2}AB}{AB} = \tfrac{1}{2};$$

or, since 30° and 60° are complements the one of the other,

$$\sin 30° = \cos 60° = \tfrac{1}{2}, \tag{21}$$

whence by (10)

$$\cos 30° = \sin 60° = \sqrt{1 - \tfrac{1}{4}} = \tfrac{1}{2}\sqrt{3}. \tag{22}$$

Then, by (13) and (5),

$$\tan 30° = \cot 60° = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}, \tag{23}$$

$$\cot 30^\circ = \tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}, \tag{24}$$

$$\sec 30^\circ = \operatorname{cosec} 60^\circ = \frac{1}{\frac{1}{2}\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}, \tag{25}$$

$$\operatorname{cosec} 30^\circ = \sec 60^\circ = \frac{1}{\frac{1}{2}} = 2. \tag{26}$$

72. *To find the* SINE, *&c. of* 45°.

Since 45° is the complement of 45°,

$$\sin 45^\circ = \cos 45^\circ.$$

Then making $A = 45^\circ$ in (8), we have

$$\sin^2 45^\circ + \cos^2 45^\circ = 2 \sin^2 45^\circ = 2 \cos^2 45^\circ = 1,$$

or

$$\sin^2 45^\circ = \cos^2 45^\circ = \tfrac{1}{2},$$

$$\sin 45^\circ = \cos 45^\circ = \sqrt{\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{2}. \tag{27}$$

Hence by (13) and (5),

$$\tan 45^\circ = \cot 45^\circ = \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = 1, \tag{28}$$

$$\sec 45^\circ = \operatorname{cosec} 45^\circ = \frac{1}{\frac{1}{2}\sqrt{2}} = \sqrt{2}. \tag{29}$$

73. *To find the* SINE, *&c. of* 0° *and of* 90°.

Since 0° and 90° are complements the one of the other,

$$\sin 0^\circ = \cos 90^\circ.$$

Then making $a = b$ in (19) and (20), we have

$$\sin 0^\circ = \cos 90^\circ = \sin a \cos a - \cos a \sin a = 0, \tag{30}$$

$$\cos 0^\circ = \sin 90^\circ = \cos a \cos a + \sin a \sin a,$$

or by (8), $\cos 0^\circ = \sin 90^\circ = \cos^2 a + \sin^2 a = 1.$ (31)

Hence by (13) and (5),

$$\tan 0^\circ = \cot 90^\circ = \tfrac{0}{1} = 0, \tag{32}$$

$$\cot 0^\circ = \tan 90^\circ = \tfrac{1}{0} = \infty, \tag{33}$$

$$\sec 0^\circ = \operatorname{cosec} 90^\circ = \tfrac{1}{1} = 1, \tag{34}$$

$$\operatorname{cosec} 0^\circ = \sec 90^\circ = \tfrac{1}{0} = \infty. \tag{35}$$

74. *To find the* SINE, *&c. of* 180°.

Let $a = b = 90°$ in (17) and (18); then, by means of (30) and (31), we have

$$\sin 180° = 1 \times 0 + 0 \times 1 = 0, \tag{36}$$

$$\cos 180° = 0 \times 0 - 1 \times 1 = -1. \tag{37}$$

Hence, by (13) and (5),

$$\tan 180° = \frac{0}{-1} = 0, \qquad \cot 180° = \frac{1}{0} = \infty, \tag{38}$$

$$\sec 180° = \frac{1}{-1} = -1, \quad \operatorname{cosec} 180° = \frac{1}{0} = \infty. \tag{39}$$

75. *To find the* SINE, *&c. of* 270°.

Let $a = 180°$ and $b = 90°$ in (17) and (18), and we have

$$\sin 270° = 0 \times 0 + (-1) \times 1 = -1, \tag{40}$$

$$\cos 270° = (-1) \times 0 - 0 \times 1 = 0. \tag{41}$$

Hence, by (13) and (5),

$$\tan 270° = \frac{-1}{0} = \infty, \qquad \cot 270° = \frac{0}{-1} = 0, \tag{42}$$

$$\sec 270° = \frac{1}{0} = \infty, \quad \operatorname{cosec} 270° = \frac{1}{-1} = -1. \tag{43}$$

76. *To find the* SINE, *&c. of* 360°.

Let $a = b = 180°$ in (17) and (18), and we have

$$\sin 360° = 0 \times (-1) + (-1) \times 0 = 0, \tag{44}$$

$$\cos 360° = (-1) \times (-1) - 0 \times 0 = 1. \tag{45}$$

But these values for the sine and cosine of 360° are the same as those for the sine and cosine of 0°. Hence,

All the trigonometric functions of 360° *are the same as those for* 0°.

77. *To find the* SINE, *&c. of the supplement of an angle.*

Let $a = 180°$ in (19) and (20); then, by means of (36) and (37), we have

$$\sin (180° - b) = \sin b, \quad \cos (180° - b) = -\cos b \tag{46}$$

whence, by (13) and (5),

$$\tan(180^\circ - b) = \frac{\sin b}{-\cos b} = -\tan b, \tag{47}$$

$$\cot(180^\circ - b) = \frac{1}{-\tan b} = -\cot b, \tag{48}$$

$$\sec(180^\circ - b) = \frac{1}{-\cos b} = -\sec b, \tag{49}$$

$$\operatorname{cosec}(180^\circ - b) = \frac{1}{\sin b} = \operatorname{cosec} b; \tag{50}$$

that is, *the sine and cosecant of the supplement of an angle are the same as those of the angle itself; and the cosine, tangent, cotangent, and secant are the negatives of those of the angle.*

78. It follows from the preceding article, that *the sine and cosecant of an obtuse angle are positive, while its cosine, tangent, cotangent, and secant are negative,* as has before been shown geometrically (Art. 68, 69).

79. *To find the* SINE, *&c. of a negative angle.*

Let $a = 0^\circ$ in (19) and (20); then, by means of (30), (31), (13), and (5), we have

$$\sin(-b) = -\sin b, \qquad \cos(-b) = \cos b, \tag{51}$$

$$\tan(-b) = -\tan b, \qquad \cot(-b) = -\cot b, \tag{52}$$

$$\sec(-b) = \sec b, \qquad \operatorname{cosec}(-b) = -\operatorname{cosec} b; \tag{53}$$

that is, *the cosine and secant of the negative of an angle are the same as those of the angle itself; and the sine, tangent, cotangent, and cosecant of the negative of an angle are the negatives of those of the angle.* These results correspond with those obtained geometrically (Art. 68).

80. *To find the* SINE, *&c. of an angle which exceeds* 180°.

Let $a = 180^\circ$ in (17) and (18); then, by means of (36) and (37), we have

$$\sin(180^\circ + b) = -\sin b, \qquad \cos(180^\circ + b) = -\cos b, \tag{54}$$

$$\tan(180^\circ + b) = \tan b, \qquad \cot(180^\circ + b) = \cot b, \tag{55}$$

$$\sec(180^\circ + b) = -\sec b, \qquad \operatorname{cosec}(180^\circ + b) = -\operatorname{cosec} b; \tag{56}$$

that is, *the tangent and cotangent of an angle which exceeds* 180° *are equal to those of its excess above* 180°; *and the sine, cosine, secant, and cosecant of this angle are the negatives of those of its excess.*

81. It follows from the preceding article, that *the tangent and cotangent of an angle which exceeds* 180° *and is less than* 270° *are positive; while its sine, cosine, secant, and cosecant are negative.*

Also, by considering b greater than 90° (Art. 78), that *the cosine and secant of an angle which exceeds* 270° *and is less than* 360° *are positive; while its sine, tangent, cotangent, and cosecant are negative.* (See Art. 68, 69.)

82. *To find the* SINE, *&c. of an angle which exceeds* 360°.

Let $a = 360°$ in (17) and (18); then, by means of (44) and (45), we have

$$\sin(360° + b) = \sin b, \quad \cos(360° + b) = \cos b; \quad (57)$$

that is, *all the trigonometric functions of an angle which exceeds* 360° *are the same as those of the excess above* 360°, *so that* 360° *may be suppressed as often as it can be, so far as the function of the angle is concerned.*

83. The trigonometric functions of any angle whatever can now be readily expressed in those of an angle not exceeding 90°. Thus,

1. The trigonometric functions of any *negative* angle can be made to depend upon those of the corresponding *positive* angle (Art. 79).

2. Any angle exceeding 360°, as far as the trigonometric functions are concerned, may be replaced by an angle less than 360° (Art. 82).

3. Any angle exceeding 180° can in like manner be replaced by an angle less than 180° (Art. 80).

4. The trigonometric functions of any angle exceeding 90° may be made to depend upon those of an angle less than 90° (Art 77, 78).

For example,

$$\sin 600° = \sin(360° + 240°) = \sin 240° = \sin(180° + 60°)$$
$$= -\sin 60°,$$

$$\tan(-1000°) = -\tan 1000° = -\tan(720° + 280°) =$$
$$-\tan 280° = -\tan(180° + 100°) = -\tan 100° =$$
$$-\tan(180° - 80°) = \tan 80°.$$

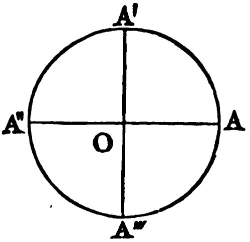
read 84. It will be seen from the preceding articles that the sine and cosine may have any value between -1 and $+1$; the tangent and cotangent, any value between $-\infty$ and $+\infty$; the secant and cosecant, any value between $-\infty$ and -1 and between $+1$ and $+\infty$. It might also be shown that the versed sine, coversed sine, and suversed sine may have any value between 0 and 2.

It will also be found that no trigonometric function changes its sign except when it passes through the value zero or the value infinity.

X

read 85. The values of the functions of 0°, 90°, 180°, 270°, and 360° can be readily recalled, by being represented geometrically according to the definitions of Art. 54. Thus,

$\sin 0° = 0$, $\cos 0° = OA = R = 1$,
$\tan 0° = 0$, and $\sec 0° = OA = 1$;
$\sin 90° = OA' = 1$, $\cos 90° = 0$;
$\sin 180° = 0$, $\cos 180° = OA'' = -1$;
$\sin 270° = OA''' = -1$, $\cos 270° = 0$.



The tangent for either 90° or 270° would be a line drawn through A parallel to the secant, which would be $A'A'''$ prolonged, and as they are each to be limited by their mutual intersection, they must both be infinite.

The cotangent of either 0° or 180° would be an infinite line drawn through A', and parallel to the cosecant, which would be AA'' infinitely prolonged.

$\text{vers } 0° = 0$, $\text{vers } 90° = \text{vers } 270° = AO = 1$, $\text{vers } 180° = AA'' = 2$; $\text{covers } 0° = \text{covers } 180° = A'O = 1$, $\text{covers } 90° = 0$, $\text{covers } 270° = A'A''' = 2$; $\text{suvers } 0° = A''A = 2$, $\text{suvers } 90° = \text{suvers } 270° = A''O = 1$, $\text{suvers } 180° = 0$.

TABLE.

Degrees.	Sine.	Cosine.	Tangent.	Cotangent.	Secant.	Cosecant.
0	0	+1	0	∞	+1	∞
90	+1	0	∞	0	∞	+1
180	0	—1	0	∞	—1	∞
270	—1	0	∞	0	∞	—1
360	0	+1	0	∞	+1	∞

GENERAL FORMULÆ.

86. From the four fundamental formulæ (Art. 64), a large number of other formulæ of general utility may be deduced.

87. *To find expressions for the products of sines and cosines, and for their sums and differences.*

The sum and difference of equations (17) and (19) are

$$\sin(a+b)+\sin(a-b)=2\sin a\cos b, \qquad (58)$$

$$\sin(a+b)-\sin(a-b)=2\cos a\sin b; \qquad (59)$$

and the sum and difference of (18) and (20) are

$$\cos(a+b)+\cos(a-b)=2\cos a\cos b, \qquad (60)$$

$$\cos(a+b)-\cos(a-b)=-2\sin a\sin b. \qquad (61)$$

Now, if in the formulæ we let

$$a+b=A, \text{ and } a-b=B,$$

that is,

$$a=\tfrac{1}{2}(A+B), \text{ and } b=\tfrac{1}{2}(A-B),$$

we shall have,

$$\sin A+\sin B=2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B), \qquad (62)$$

$$\sin A-\sin B=2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B), \qquad (63)$$

$$\cos A+\cos B=2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B), \qquad (64)$$

$$\cos B-\cos A=2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B), \qquad (65)$$

in which A and B represent any two angles, and consequently admit of every possible value. These formulæ are of frequent application, especially in calculations effected by logarithms; (58), (59), (60), and (61) serve to transform a product to a sum or difference, and (62), (63), (64), and (65) serve to transform a sum or difference to a product.

88. Dividing formula (62) by (63), we have

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)},$$

which, by means of (13) and (5), becomes

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \tfrac{1}{2}(A+B) \cot \tfrac{1}{2}(A-B),$$

or,

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}, \qquad (66)$$

and by (14),

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\cot \frac{1}{2}(A-B)}{\cot \frac{1}{2}(A+B)}; \qquad (67)$$

that is,

The sum of the sines of two angles is to their difference as the tangent of half the sum of the angles is to the tangent of half their difference, or as the cotangent of half their difference is to the cotangent of half their sum.

89. By means of (62), (63), (64), and (13), we obtain,

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)} = \tan \tfrac{1}{2}(A+B), \qquad (68)$$

$$\frac{\sin A - \sin B}{\cos A + \cos B} = \frac{2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)}{2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)} = \tan \tfrac{1}{2}(A-B), \qquad (69)$$

that is,

The sum of the sines of two angles divided by the sum of their cosines is equal to the tangent of half the sum of the angles; and

The difference of the sines of two angles divided by the sum of their cosines is equal to the tangent of half the difference of the angles.

90. *To find the tangent and cotangent of the sum of two angles by means of their tangents.*

Let A and B be the two angles; then, by (13),

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)};$$

or, substituting for $\sin (A+B)$, $\cos (A+B)$, their values from (17) and (18),

$$\tan (A+B)=\frac{\sin A \cos B+\cos A \sin B}{\cos A \cos B-\sin A \sin B}.$$

Dividing all the terms of the numerator and denominator by $\cos A \cos B$, we have

$$\tan (A+B)=\frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{1-\frac{\sin A}{\cos A}\times\frac{\sin B}{\cos B}},$$

or, by (13),

$$\tan (A+B)=\frac{\tan A+\tan B}{1-\tan A \tan B}, \qquad (70)$$

and, by (5),

$$\cot (A+B)=\frac{1-\tan A \tan B}{\tan A+\tan B}. \qquad (71)$$ omit

91. *To find the tangent and cotangent of the difference of two angles by means of their tangents.*

By (13),

$$\tan (A-B)=\frac{\sin (A-B)}{\cos (A-B)},$$

then, in like manner as in Art. 90, we have,

$$\tan (A-B)=\frac{\tan A-\tan B}{1+\tan A \tan B}, \qquad (72)$$

and

$$\cot (A-B)=\frac{1+\tan A \tan B}{\tan A-\tan B}. \qquad (73)$$ omit

92. *To find the* SINE, *&c. of double an angle, by means of the functions of the angle itself.*

In the expression for $\sin (A+B)$ and $\cos (A+B)$, let $B=A$,

then $\qquad \sin 2A=\sin A \cos A+\sin A \cos A,$

or, $\qquad \sin 2A=2\sin A \cos A; \qquad (74)$

and $\qquad \cos 2A=\cos A \cos A-\sin A \sin A,$

or, $\qquad \cos 2A=\cos^2 A-\sin^2 A. \qquad (75)$

$\sin^2 A = 1 - \cos^2 A$

$\cos 2A = \cos^2 A - 1 + \cos^2 A$

Substituting in the latter, first the value of $\cos^2 A$ and then the value of $\sin^2 A$, from (9) and (11), we have

$$\cos 2A = 1 - 2\sin^2 A, \tag{76}$$

$$\cos 2A = 2\cos^2 A - 1. \tag{77}$$

By means of (70) and (71), we have

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}, \tag{78}$$

$$\cot 2A = \frac{1 - \tan^2 A}{2\tan A}. \tag{79}$$

93. *To find the* SINE, *&c. of half an angle by means of the sine or cosine of the angle itself.*

Let $\frac{1}{2}A = A$ in (76) and (77), and transpose; then

$$2\sin^2 \tfrac{1}{2}A = 1 - \cos A, \tag{80}$$

$$2\cos^2 \tfrac{1}{2}A = 1 + \cos A; \tag{81}$$

whence

$$\sin \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{2}}, \quad \cos \tfrac{1}{2}A = \sqrt{\frac{1 + \cos A}{2}}, \tag{82}$$

$$\tan \tfrac{1}{2}A = \frac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}. \tag{83}$$

By multiplying both numerator and denominator by $\sqrt{1 - \cos A}$, or by $\sqrt{1 + \cos A}$, we obtain (12),

$$\tan \tfrac{1}{2}A = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}. \tag{84}$$

NATURAL SINES AND COSINES.

94. NATURAL SINES, COSINES, &c. are the values of the sines, cosines, &c. expressed in natural numbers.

95. A table containing these values is called a table of natural sines and cosines.

96. The semi-circumference of a circle whose radius is 1 is equal to 3.1415926 nearly (Geom., Prop. XV. Sch. 2, Bk. VI.),

and this divided by 10800, the number of minutes in 180°, will give .0002908882 for the arc of 1′, which may be taken also for the sine of an angle of 1′.

By means of formula (10),

$$\cos 1' = \sqrt{1 - \sin^2 1'} = .9999999577.$$

Then by transposition of formulæ (58) and (60),

$$\sin (a + b) = 2 \sin a \cos b - \sin (a - b),$$
$$\cos (a + b) = 2 \cos a \cos b - \cos (a - b),$$

in which, making b equal to 1′, and a, in succession, equal 1′, 2′, 3′, &c., we obtain for the sines,

$$\sin 2' = 2 \sin 1' \cos 1' - \sin 0' = .0005817764,$$
$$\sin 3' = 2 \sin 2' \cos 1' - \sin 1' = .0008726646,$$
$$\sin 4' = 2 \sin 3' \cos 1' - \sin 2' = .0011635526,$$
$$\text{\&c.,} \qquad\qquad \text{\&c.,}$$

and for the cosines,

$$\cos 2' = 2 \cos 1' \cos 1' - \cos 0' = .9999998308,$$
$$\cos 3' = 2 \cos 2' \cos 1' - \cos 1' = .9999996193,$$
$$\cos 4' = 2 \cos 3' \cos 1' - \cos 2' = .9999993232,$$
$$\text{\&c.,} \qquad\qquad \text{\&c.,}$$

thus obtaining the sines and cosines up to 45°.

The tangents may readily be found by dividing the sines by the cosines (13); and the secants, cotangents, and cosecants by dividing 1 by the cosines, tangents, and sines, respectively (Art. 51).

97. The sines, tangents, and secants of angles greater than 45° are respectively the cosines, cotangents, and cosecants of their complements, which are less than 45°; and, by their definitions, cosines, cotangents, and cosecants are the sines, tangents, and secants of complements (Art. 50). Thus,

$$\sin 46° = \cos (90° - 46°) = \cos 44°, \quad \tan 51° = \cot 39°,$$
$$\cos 50° = \sin 40, \quad \cot 88° = \tan 2°.$$

Tables, therefore, do not go beyond 45°; or, rather, are so arranged that each number answers as a function of both an angle less than 45° and its complement greater than 45°.

TABLE OF LOGARITHMIC SINES, COSINES, &c.

98. A TABLE of LOGARITHMIC SINES, COSINES, &c. contains the logarithms of the numbers expressing the natural sines, cosines, &c.

99. Since the sines and cosines are never greater than 1, and tangents likewise, when under 45°, their logarithms properly have *negative* characteristics. But to avoid the inconvenience of these, the characteristics are, by common consent, increased by 10. Thus the characteristic 9 is used in the place of —1, 8 in place of —2, &c.

The radius, therefore, of the logarithmic sines, cosines, &c. is, as arbitrarily assumed, 10^{10}, or 10,000,000,000.

100. In the accompanying table the degrees are given at the top and bottom of the page, and the minutes in the columns at the sides designated by M.

The column headed D contains the increase or decrease for 1 second. This is obtained by taking one sixtieth of the difference between the logarithmic sine, cosine, &c. of an angle or arc, and that next exceeding it by 1 minute. The result is placed against the lesser angle or arc.

To find the Logarithmic Sine, &c. of any Angle or Arc.

101. If the angle or arc is less than 45°, look for the degrees at the *top* of the table, and for the minutes on the *left;* then, opposite to the minutes, on the same horizontal line, and in the column headed *Sine,* will be found the logarithmic sine; in the column headed *Cosine* will be found the logarithmic cosine, &c. Thus,

the logarithmic sine of 19° 23′ is 9.520990,
" " cosine of 31° 47′ " 9.929442,
" " tangent of 43° 5′ " 9.970922.

102. If the angle or arc is between 45° and 90°, look for the degrees at the *bottom* of the table, and for the minutes on the *right;* then, opposite to the minutes, and in the column designated at the bottom *Sine,* will be found the logarithmic sine; in the column designated at the bottom *Cosine* will be found the logarithmic cosine, &c. Thus,

the logarithmic sine of 80° 11′ is 9.993594,
" " cosine of 65° 59′ " 9.609597,
" " cotangent of 73° 35′ " 9.469280.

103. If the angle or arc is between 90° and 180°, subtract it from 180°, and take the logarithmic sine, &c. of the remainder. Thus,

the logarithmic sine of 112° is the logarithmic sine of 68°.
" " tangent of 98° " " tangent of 82°.

104. If the angle or arc is expressed in degrees, minutes, and seconds, find the logarithmic sine, &c. of the degrees and minutes as before; then multiply the number opposite, in the column headed D, by the seconds, and *add* the product to the number first found, for sines and tangents, but *subtract* it for cosines and cotangents.

Thus, if the logarithmic sine of 30° 25′ 42″ is required,

The logarithmic sine of 30° 25′ is		9.704395
Tabular difference,	3.59	
Number of seconds,	42	
Product,	150.78	150.78
Logarithmic sine of 30° 25′ 42″ is		9.704546

It is customary to omit the decimal figures at the right, but to increase the last figure retained, by 1, when the figure at the left of those omitted is 5 or greater than 5.

105. The secants and cosecants are not included in the table, since they may be readily derived from the cosines and sines.

By (5), $\sec A \cos A = 1$, and $\log \sec A + \log \cos A = 0$; but as log sec and log cos are each increased by 10 (Art. 99), the second member of the equation must be increased by 20, that is, logarithmic secant = 20 — logarithmic cosine.

In like manner,

logarithmic cosecant = 20 — logarithmic sine.

Hence, to find the logarithmic *secant*, subtract the logarithmic cosine from 20; and to find the logarithmic *cosecant*, subtract the logarithmic sine from 20. Thus,

The logarithmic secant of 65° 59′ is 10.390403
" " cosecant of 30° 25′ 42″ " 10.295454.

TO FIND THE ANGLE OR ARC CORRESPONDING TO ANY LOGARITHMIC SINE, &C.

106. Look in the column designated by the same name with the given logarithm for the sine, &c. which is *nearest* to the given one, and if the name be at the *head* of the column, take the degrees at the *top* of the table, and the minutes on the *left*; but if the name be at the *foot* of the column, take the degrees at the *bottom*, and the minutes on the *right*. Thus,

The angle or arc corresponding to the logarithmic sine 9.681443 is 28° 42′.

The angle or arc corresponding to the logarithmic tan 9.984079 is 43° 57′.

The angle or arc corresponding to the logarithmic cos 9.731603 is 57° 23′.

107. If the given logarithmic sine, &c. is not found exactly, or very nearly, then, to find the *seconds*, subtract from the given logarithm that next less in the table, to the remainder annex two ciphers, divide the result by the number in the column headed D, and the quotient will be the number of seconds to be *added* to the degrees and minutes of the lesser logarithm for sines and tangents, or to be *subtracted* for cosines and cotangents.

Thus, to find the angle or arc corresponding to the logarithmic sine 9.938070.

Given log sine,	9.938070		
Next less,	9.938040	corresponding angle,	60° 7′
Diff. from column D,	1.21)30.00		25″

The log sine 9.938070 has for its cor. angle or arc, 60° 7′ 25″.

The angle or arc corresponding to the logarithmic tangent 9.497200 is 17° 26′ 33″.

The angle or arc corresponding to the logarithmic cosine 9.792477 is 51° 40′ 30″.

EXAMPLES.

1. Required the logarithmic sine of 28° 42′. Ans. 9.681443.
2. Required the logarithmic cosine of 59° 33′ 47″.
Subtract from 180° Ans. 9.704657.
3. Required the logarithmic cotangent of 127° 2′.
Ans. 9.877640.
4. Required the logarithmic sine of 81° 20′. Ans. 9.995013.
5. Required the logarithmic secant of 51° 40′ 30″.
Ans. 10.207523.
6. Required the logarithmic tangent of 74° 21′ 20″.
Ans. 10.552778.
7. Required the logarithmic cosecant of 102° 24′ 41″.
Ans. 10.010270.
8. Required the logarithmic tangent of 1° 59′ 51″.8.
Ans. 8.542587.
9. Required the angle of the logarithmic sine 9.999969.
Ans. 89° 19′.
10. Required the arc of the logarithmic tangent 9.645270.
Ans. 23° 50′ 17″.
11. Required the angle of the logarithmic cosine 9.598075.
Ans. 66° 39′.
12. Required the angle of the logarithmic cotangent 10.301470.
Ans. 26° 32′ 31″.
13. Required the arc of the logarithmic sine 9.893410.
Ans. 51° 28′ 40″.
14. Required the angle of the logarithmic cosine 9.421157.
74° 42′ 31″ Ans. 105° 17′ 29″.
15. Required the arc of the logarithmic tangent 9.692125.
Ans. 26° 12′ 20″.
16. Required the angle of the logarithmic cotangent 9.421901.
Ans. 75° 12′ 6″.

BOOK III.

SOLUTION OF PLANE TRIANGLES.

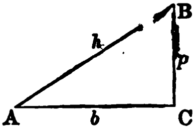
108. **The solution of triangles** is the process by which, when the values of a sufficient number of their elements are given, the values of the remaining elements are computed.

The *elements* of every triangle are the three sides and the three angles. Three of these elements must be given, one of which must be a side, in order to solve a plane triangle.

The solution of plane triangles depends upon the following

FUNDAMENTAL PROPOSITIONS.

109. *In a right-angled triangle, the side opposite to an acute angle is equal to the product of the hypothenuse into the sine of the angle; and the side adjacent to an acute angle is equal to the product of the hypothenuse into the cosine of the angle.*



Let $A\,B\,C$ be a triangle having a right angle at C; then, by (1),

$$\sin A = \frac{p}{h}, \qquad \sin B = \frac{b}{h};$$

therefore $\quad p = h \sin A, \qquad b = h \sin B. \qquad (85)$

Again, by (4), $\quad \cos A = \frac{b}{h}, \qquad \cos B = \frac{p}{h};$

therefore $\quad b = h \cos A, \qquad p = h \cos B. \qquad (86)$

110. *In a right-angled triangle, the side opposite to an acute angle is equal to the product of the other side into the tangent of the angle; and the side adjacent to an acute angle is equal to the product of the other side into the cotangent of the angle.*

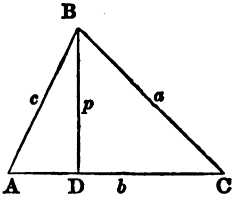
For, by (2), $\tan A = \frac{p}{b}$, $\tan B = \frac{b}{p}$,

therefore $p = b \tan A$, $b = p \tan B$. (87)

Again, by (4), $\cot A = \frac{b}{p}$, $\cot B = \frac{p}{b}$,

therefore $b = p \cot A$, $p = b \cot B$. (88)

111. *In any plane triangle, the sides are proportional to the sines of the opposite angles.*



Let $A\,B\,C$ be any triangle, in which the sides opposite the angles A, B, C, respectively, are denoted by a, b, and c. From one of the angles, as B, draw $B\,D$ perpendicular to the opposite side $A\,C$, and denote the line $B\,D$ by p. Then the right-angled triangles, $C\,B\,D$, $A\,B\,D$, give, by (85),

$$p = a \sin C, \qquad p = c \sin A;$$

whence, $a \sin C = c \sin A$,

which gives the proportion

$$a : c :: \sin A : \sin C. \tag{89}$$

In like manner it may be proved that

$$a : b :: \sin A : \sin B, \tag{90}$$

$$c : b :: \sin C : \sin B, \tag{91}$$

and these three proportions give

$$a : b : c :: \sin A : \sin B : \sin C, \tag{92}$$

which may also be written

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}. \tag{93}$$

The angle C was *acute*, but had it been *obtuse*, or a right angle, the results would have been the same. The proposition, therefore, applies in every case.

112. *In any plane triangle, the sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.*

For, by (90), $a : b :: \sin A : \sin B$;

whence (Geom., Prop. XII. Bk. II.),

$$a + b : a - b :: \sin A + \sin B : \sin A - \sin B,$$

which may also be written,

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}.$$

But, by formula (66),

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)};$$

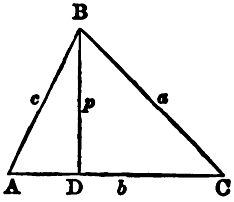
therefore,
$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}, \tag{94}$$

or, as it may be written,

$$a + b : a - b :: \tan \tfrac{1}{2}(A+B) : \tan \tfrac{1}{2}(A-B). \tag{95}$$

113. *In any triangle, the square of any side is equal to the sum of the squares of the two other sides, diminished by twice the rectangle of these sides multiplied by the cosine of the included angle.*

Let ABC be any plane triangle, in which the sides opposite the angles A, B, C, respectively, are denoted by a, b, c. Draw BD from one of the angles, B, perpendicular to the opposite side, AC. Then, if A is acute, we have by Geometry (Prop. XII. Bk. IV.),

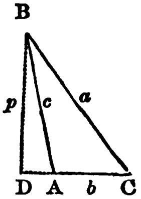


$$a^2 = b^2 + c^2 - 2\,b\,AD;$$

but from the right-angled triangle ABD, by (86), we have

$$AD = c \cos A;$$

therefore,
$$a^2 = b^2 + c^2 - 2\,b\,c \cos A. \tag{96}$$



When the angle A is obtuse, the point D will fall on the other side of A, and we have by Geometry (Prop. XIII. Bk. IV.),

$$a^2 = b^2 + c^2 + 2\,b\,AD.$$

But since BAD is now the supplement of BAC, by Art. 77 we have

$$AD = c \cos BAD = -c \cos BAC = -c \cos A.$$

Substituting this value of AD, we have as before,

$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

When A is a right angle and a the hypothenuse, $\cos A$ is zero (30), and (96) becomes

$$a^2 = b^2 + c^2,$$

and thus the formula (96) is true, whatever the angle A may be.

In like manner we have

$$b^2 = a^2 + c^2 - 2\,ac \cos B, \tag{97}$$

$$c^2 = a^2 + b^2 - 2\,ab \cos C. \tag{98}$$

114. *The cosine of any angle of a plane triangle is equal to the fraction whose numerator is the sum of the squares of the containing sides, diminished by the square of the opposite side, and whose denominator is twice the product of the containing sides.*

For, by (96), $a^2 = b^2 + c^2 - 2\,bc \cos A$,

whence,
$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}. \tag{99}$$

Similarly, from (97) and (98), we have

$$\cos B = \frac{a^2 + c^2 - b^2}{2\,ac}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2\,ab}. \tag{100}$$

115. By these formulæ the angles of a triangle can be found when the sides are given, but they cannot be conveniently applied in computation by logarithms.

We then subtract both members of formula (99) from 1, and obtain

$$1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2\,bc},$$

and, substituting for $1 - \cos A$ its value, $2 \sin^2 \frac{1}{2} A$, by (80), we have

$$2 \sin^2 \tfrac{1}{2} A = 1 - \frac{b^2 + c^2 - a^2}{2\,b\,c} = \frac{a^2 - (b - c)^2}{2\,b\,c}$$

$$= \frac{(a + b - c)\ (a - b + c)}{2\,bc};$$

whence, $$\sin^2 \tfrac{1}{2} A = \frac{(a + b - c)\ (a - b + c)}{4\,bc}. \qquad (101)$$

Let now $2\,s = a + b + c$, so that s is half the sum of the sides of the triangle; then

$$a + b - c = 2\,(s - c), \quad a - b + c = 2\,(s - b).$$

Substituting these values in the preceding equation, and reducing, we have

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)\ (s - c)}{b\ c}}. \qquad (102)$$

In like manner we may obtain

$$\sin \tfrac{1}{2} B = \sqrt{\frac{(s - a)\ (s - c)}{a\ c}}, \qquad (103)$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{(s - a)\ (s - b)}{a\ b}}. \qquad (104)$$

That is,

The sine of half of any angle in a plane triangle is equal to the square root of half the sum of the three sides less one of the adjacent sides, into half the sum less the other adjacent side, divided by the rectangle of the two adjacent sides.

116. If 1 be added to both sides of (99), then, substituting for $1 + \cos A$ its value, $2 \cos^2 \frac{1}{2} A$, by (81), we have

$$2 \cos^2 \tfrac{1}{2} A = 1 + \frac{b^2 + c^2 - a^2}{2\,b\,c} = \frac{(b + c)^2 - a^2}{2\,b\,c}$$

$$= \frac{(b + c + a)\ (b + c - a)}{2\ b\ c};$$

whence, $$\cos^2 \tfrac{1}{2} A = \frac{(b + c + a)\ (b + c - a)}{4\,bc}. \qquad (105)$$

Let now s = half the sum of the sides of the triangle, as in Art. 115; then,

$$b + c + a = 2s, \quad b + c - a = 2(s - a).$$

Substituting these values in the preceding equation, we have

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}. \qquad (106)$$

Similarly,

$$\cos \tfrac{1}{2} B = \sqrt{\frac{s(s-b)}{ac}}, \qquad (107)$$

$$\cos \tfrac{1}{2} C = \sqrt{\frac{s(s-c)}{ab}}. \qquad (108)$$

That is,

The cosine of half of any angle of a plane triangle is equal to the square root of half the sum of the three sides, into half the sum less the side opposite the angle, divided by the rectangle of the two adjacent sides.

117. Dividing (102) by (106), (103) by (107), and (104) by (108), we have, by (13),

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}; \qquad (109)$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}; \qquad (110)$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}. \qquad (111)$$

That is,

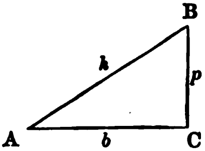
The tangent of half of any angle of a plane triangle is equal to the square root of half the sum of the three sides, less one of the adjacent sides, into half the sum less the other adjacent side, divided by half the sum, into half the sum less the side opposite the angle.

SOLUTION OF RIGHT-ANGLED TRIANGLES.

118. In a right-angled triangle, the side opposite to the right angle is called the *hypothenuse;* that adjacent to the right angle, and upon which the triangle is supposed to stand, is called the

base; and the other side adjacent to the right angle, the *perpendicular.* The base and perpendicular have been termed the sides about the right angle. Of the acute angles, that adjacent to the base has been termed the angle at the base, and the other the angle at the perpendicular.

Thus, let $A B C$ be any right-angled triangle, with the right angle at C, then h represents the hypothenuse, b the base, p the perpendicular, A the acute angle at the base, and B the acute angle at the perpendicular.



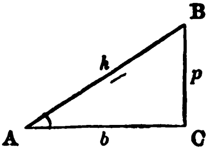
119. In order to solve the triangle, two elements other than the right angle must be given, one of them being a side. Hence there will be four cases in which there may be given, respectively,

I. The hypothenuse and an acute angle.
II. A side about the right angle and an acute angle.
III. The hypothenuse and a side about the right angle.
IV. The two sides about the right angle.

CASE I.

120. *Given the hypothenuse and an acute angle.*

Let there be given, in the right-angled triangle $A B C$, the hypothenuse h and the acute angle A; to find the angle B, the perpendicular p, and the base b.



To find B. The angle B is the complement of A (Art. 44); hence,

$$B = 90° - A.$$

To find p and b. By (85) and (86) we have

$$p = h \sin A = h \cos B,$$

$$b = h \cos A = h \sin B;$$

or, by logarithms,

$$\log p = \log h + \log \sin A = \log h + \log \cos B, \qquad (112)$$

$$\log b = \log h + \log \cos A = \log h + \log \sin B. \qquad (113)$$

That is,

The logarithm of either side about the right angle is equal to the logarithm of the hypothenuse, plus the logarithmic sine of the opposite angle, or plus the logarithmic cosine of the adjacent angle.

NOTE 1. As the logarithmic sine and cosine are increased by 10 (Art. 99), the resulting logarithm will be so much too great, and must be diminished by 10. This increase by 10 will affect the work wherever the logarithms of trigonometric functions are used.

NOTE 2 The last figure of an answer may occasionally be found to differ from the one given in this work, when it has been obtained by the use of different formulæ or tables. The results are not, however, generally carried so far as to admit of such a difference. When two methods of solving give different results, that is inserted which is most accurate, whether obtained by the usual method or not.

EXAMPLES.

1. Given the hypothenuse of a right-angled triangle equal to 1785.395 feet, and the angle at the base equal to 59° 37′ 42″; to solve the triangle.

Solution. The angle at the perpendicular $= 90° - 59° 37' 42'' = 30° 22' 18''$. Let, now, $h = 1785.395$ feet and $A = 59° 37' 42''$, and we have, by (112) and (113),

$h = 1785.395$	log 3.251734		log 3.251734
$A = 59° 37' 42''$	log sin 9.935892		log cos 9.703813
$p = 1540.37$	log 3.187626	$b = 902.708$	log 2.955547

Ans. Angle at the perpendicular, 30° 22′ 18″; perpendicular, 1540.37 feet; base, 902.708 feet.

2. Given the hypothenuse of a right-angled triangle equal to 25 yards, and one of the acute angles equal to 54° 30′; to solve the triangle.

3. Given the hypothenuse of a right-angled triangle equal to 173.2 feet, and one of the acute angles equal to 37° 2′ 43″; required the other parts.

Ans. Angle, 52° 57′ 17″; sides, 104.34 feet and 138.24 feet.

CASE II.

121. *Given a side about the right angle, and an acute angle.*

Let there be given (Fig. Art. 120) the side b and the angle A; to solve the triangle.

To find B. The angle B is the complement of A; hence,

$$B = 90^\circ - A.$$

To find p. By (87) and (88), we have

$$p = b \tan A = b \cot B;$$

or, by logarithms,

$$\log p = \log b + \log \tan A = \log b + \log \cot B. \quad (114)$$

To find h. By means of (113), we obtain

$$\log h = \log b - \log \cos A = \log b - \log \sin B. \quad (115)$$

Given the side p and the angle A; to solve the triangle.

To find B. We have, as before, the angle B, the complement of A, or $B = 90^\circ - A$.

To find b. By (87) and (88), we have

$$b = p \cot A = p \tan B;$$

or, by logarithms,

$$\log b = \log p + \log \cot A = \log p + \log \tan B. \quad (116)$$

To find h. By means of (112), we obtain

$$\log h = \log p - \log \sin A = \log p - \log \cos B. \quad (117)$$

That is,

The logarithm of either side about the right angle is equal to the logarithm of the other, plus the logarithmic tangent of the angle opposite, or plus the logarithmic cotangent of the angle adjacent to the former.

The logarithm of the hypothenuse is equal to the logarithm of either side about the right angle, minus the logarithmic sine of the angle opposite, or minus the logarithmic cosine of the angle adjacent to the side.

EXAMPLES.

1. Given the side b of a right-angled triangle equal to 902.708 feet, and the acute angle A equal to 59° 37′ 42″; to solve the triangle.

Solution. The angle $B = 90° - 59° 37' 42'' = 30° 22' 18''$. By (114) and (115), we have

$$\begin{array}{llll}
b = 902.708 & \log\ 2.955547 & & \log\ 2.955547 \\
A = 59° 37' 42'' & \log\tan\ \underline{10.232078} & & \text{ar. co. } \log\cos\ \underline{0.296187} \\
p = 1540.37 & \log\ 3.187625 & h = 1785.395 & \log\ 3.251734
\end{array}$$

Ans. Angle B, 30° 22′ 18″; perpendicular, 1540.37 feet; hypothenuse, 1785.395 feet.

2. Given one of the sides about the right angle of a right-angled triangle equal to 14.52 rods, and the opposite angle equal to 35° 30′; to solve the triangle.

3. Given the perpendicular of a right-angled triangle equal to 3555.4 yards, and the angle at the perpendicular equal to 33° 30′ 47″; to solve the triangle.

Ans. Angle at the base, 56° 29′ 13″; base, 2354.4 yards; hypothenuse, 4264.3 yards.

CASE III.

122. *Given the hypothenuse and a side about the right angle.*

Let there be given (Fig. Art. 120) the hypothenuse h and the side p; to solve the triangle.

To find A and B. By (1) and (4), we have

$$\sin A = \cos B = \frac{p}{h};$$

or, by logarithms,

$$\log \sin A = \log \cos B = \log p - \log h. \qquad (118)$$

To find b. By (85) and (86), we have

$$b = h \cos A = h \sin B;$$

or, by logarithms,

$$\log b = \log h + \log \cos A = \log h + \log \sin B. \qquad (119)$$

Also, by Geometry (Prop. XI. Bk. IV.), we have

$$h^2 = p^2 + b^2; \tag{120}$$

whence,
$$b^2 = h^2 - p^2 = (h + p)\ (h - p),$$

$$b = \sqrt{(h + p)\ (h - p)};$$

or, by logarithms,

$$\log b = \tfrac{1}{2} \log (h + p) + \tfrac{1}{2} \log (h - p). \tag{121}$$

That is,

The logarithmic sine of one of the acute angles, or the logarithmic cosine of the other, is equal to the logarithm of the side opposite the former angle, minus the logarithm of the hypothenuse.

The logarithm of either side about the right angle is equal to the logarithm of the hypothenuse, plus the logarithmic cosine of the angle adjacent, or plus the logarithmic sine of the angle opposite.

EXAMPLES.

1. Given the hypothenuse of a right-angled triangle equal to 1785.395 feet, and the perpendicular equal to 1540.37; to find the other parts.

Solution. By (118) and (119), we have

$p = 1540.37$	log	3.187626		
$h = 1785.395$	ar. co. log	6.748266	log	3.251734
$A = 59° 37' 42''$	log sin	9.935892	log cos A	9.703813
$B = 30° 22' 18''$	log cos			
		$b = 902.708$	log	2.955547

Ans. Base, 902.708 feet; angle at the base, 59° 37′ 42″; angle at the perpendicular, 30° 22′ 18″.

2. Given the hypothenuse of a right-angled triangle equal to 73 feet, and one of the sides equal to 55 feet; to solve the triangle.

3. Given the hypothenuse of a right-angled triangle equal to 643.7 rods, and the base equal to 473.8; to find the perpendicular and the two acute angles.

Ans. Perpendicular, 435.73 rods; acute angles, 42° 36′ 12″ 47° 23′ 48″.

CASE IV.

123. ***Given the two sides about the right angle.***

Let there be given (Fig. Art. 120) the sides p and b; to solve the triangle.

To find A and B. By (2) and (4), we have

$$\tan A = \cot B = \frac{p}{b};$$

or by logarithms,

$$\log \tan A = \log \cot B = \log p - \log b. \qquad (122)$$

To find h. By (1), we have

$$\sin A = \frac{p}{h}, \quad \text{whence } h = \frac{p}{\sin A}; \qquad (123)$$

or, by logarithms,

$$\log h = \log p - \log \sin A; \qquad (124)$$

Also, by (120),

$$h^2 = p^2 + b^2, \text{ whence } h = \sqrt{p^2 + b^2}. \qquad (125)$$

That is,

The logarithmic tangent of one of the acute angles, or the logarithmic cotangent of the other, is equal to the logarithm of the side opposite the former angle, minus the logarithm of the side adjacent.

The logarithm of the hypothenuse is equal to the logarithm of either side, minus the logarithmic sine of the angle opposite the side.

EXAMPLES.

1. Given of a right-angled triangle the side p equal to 1540.37 feet, and the side b equal to 902.708 feet; to solve the triangle.

Solution. By (122) and (124), we have

$p =$ 1540.37		log 3.187626		log 3.187626
$b =$ 902.708	ar. co.	log 7.044453	ar. co. log sin A	0.064108
$A =$ 59° 37′ 42″	log tan	10.232079		
$B =$ 30° 22′ 18″	log cot		$h =$ 1785.395	log 3.251734

Ans. Hypothenuse, 1785.395 feet; acute angles, 59° 37′ 42″, 30° 22′ 18″.

2. Given the perpendicular of a right-angled triangle equal to 65 feet, and the base equal to 72 feet; to find the hypothenuse and the two acute angles.

3. Given the perpendicular of a right-angled triangle equal to 2.269 rods, and the base equal to 126.9 rods; required the hypothenuse and the two acute angles.

Ans. Hypothenuse, 126.92 rods; acute angles, 1° 1′ 28″, 88° 58′ 32″.

SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

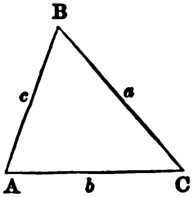
124. Since there must be given three elements, one of which is a side (Art. 108), there will be four cases, the data in them being, respectively,

I. One side and any two angles.
II. Two sides and an angle opposite one of them.
III. Two sides and the included angle.
IV. The three sides.

Case I.

125. *Given one side and two angles.*

Let there be given in the triangle ABC, the side a, and the two angles A and B; to solve the triangle.



To find C. Since the sum of the three angles must be 180°, we have

$$C = 180° - (A + B).$$

To find b and c. By (90) and (89), we have

$$a : b :: \sin A : \sin B,$$

$$a : c :: \sin A : \sin C;$$

whence,

$$b = \frac{a \sin B}{\sin A}, \qquad c = \frac{a \sin C}{\sin A}, \tag{126}$$

or, by logarithms,

$$\log b = \log a + \log \sin B - \log \sin A, \tag{127}$$

$$\log c = \log a + \log \sin C - \log \sin A. \tag{128}$$

That is,

The logarithm of the required side is equal to the logarithm of the given side, plus the logarithmic sine of the angle opposite the required side, minus the logarithmic sine of the angle opposite the given side.

EXAMPLES.

1. Given of a triangle the side a equal to 9459.31 feet, the angle A equal to $71° 3' 34''$, and the angle B equal to $53° 26'$; to find the sides b, c, and the angle C.

Solution. $C = 180° - (71° 3' 34'' + 53° 26') = 55° 30' 26''$.

Then, by (127) and (128), we have

$a = 9459.31$	log 3.975859		log 3.975859
$A = 71° 3' 34''$	ar.co.log sin 0.024176		ar. co. log sin 0.024176
$B = 53° 26'$	log sin 9.904804		
$C = 55° 30' 26''$			log sin 9.916032
$b = 8032.28$	log 3.904839	$c = 8242.64$	log 3.916067

Ans. Angle C, $55° 30' 26''$; side b, 8032.28 feet; side c, 8242.64 feet.

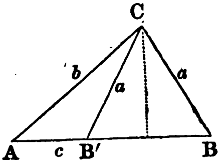
2. Given one side of a triangle equal to 110 rods, the opposite angle equal to $50° 5'$, and an adjacent angle equal to $33° 55'$; to solve the triangle.

3. Given one side of a triangle equal to 654 feet, one of the adjacent angles equal to $41° 0' 39''$, and the other adjacent angle equal to $55° 34' 8''$; to find the other parts.

Ans. Angle, $83° 25' 13''$; sides, 432 feet, 543 feet.

Case II.

126. *Given two sides and an angle opposite one of them.*



Let there be given in any triangle, ABC, the two sides a, b, and the angle A opposite to one of them; to solve the triangle.

To find B. By (90), we have

$$a : b :: \sin A : \sin B,$$

whence,

$$\sin B = \frac{b \sin A}{a}, \tag{129}$$

or, by logarithms,

$$\log \sin B = \log b + \log \sin A - \log a. \qquad (130)$$

That is,

The logarithmic sine of a required angle whose opposite side is given, is equal to the logarithm of that side, plus the logarithmic sine of the given angle, minus the logarithm of its opposite side.

To find C. We have $C = 180° - (A + B)$.

To find c. By (128), after C is found, we have

$$\log c = \log a + \log \sin C - \log \sin A.$$

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127. Whenever the given angle is acute, and the side opposite to it is less than the side adjacent to it, there may be formed, as shown in Geometry (Prob. XI. Bk. V.), two triangles, each satisfying the given conditions, and, therefore, there will be two solutions. Thus (Fig. Art. 126) with two given sides a, b, equal respectively to $C B$ and $A C$, and the given acute angle A opposite the less side $C B$, there may always be formed two triangles, $A B C$, $A B' C$, which have A, a, b in common, and the angles $A B C$, $A B' C$ supplements of each other. In one of them, therefore, the required angle is acute, and in the other it is obtuse.

When the given angle is obtuse, the required angle must of necessity be acute, since a triangle can have but one obtuse angle.

When the given angle is acute, and its opposite side is greater than the side opposite to the required angle, that must also be acute, since the greater angle must be opposite the greater side.

When the side opposite the given angle is exactly a perpendicular let fall from C on $A B$, the required angle is a right angle.

If the side opposite the given angle be less than the perpendicular, the solution is impossible, since there will be no triangle with the given parts.

When two values are admissible for B, in case of ambiguity, two corresponding values will exist for C and c.

EXAMPLES.

1. Given of any triangle $A\,B\,C$, the side b equal to 216 yards, the side a equal to 117 yards, and the angle opposite the side a equal to $22^\circ\ 37'$; to solve the triangle.

Solution. By (130), we have

$a = 117$	ar. co. log sin 7.931814
$b = 216$	log 2.334454
$A = 22^\circ\ 37'$	log sin 9.584968
$B = 45^\circ\ 13'\ 55''$ or $134^\circ\ 46'\ 5''$	log sin 9.851236
$A + B = 67^\circ\ 50\ 55''$ or $157^\circ\ 23'\ 5''$	

$$C = 180^\circ - 67^\circ\ 50'\ 55'' = 112^\circ\ 9'\ 5'', \text{ or}$$
$$C = 180^\circ - 157^\circ\ 23'\ 5'' = 22^\circ\ 36'\ 55''.$$

Then, by (128), we have

$a = 117$	log 2.068186		log 2.068186
$C = 112^\circ\ 9'\ 5''$	log sin 9.966700	or $= 22^\circ\ 36'\ 55''$	log sin 9.584943
$A = 22^\circ 37'$	ar. co. log sin 0.415032		ar. co. log sin 0.415032
$c = 281.785$	log 2.449918	or $= 116.99$	log 2.068161

Ans. Angle B, $45^\circ\ 13'\ 55''$, or $134^\circ\ 46'\ 5''$; angle C, $112^\circ\ 9'\ 5''$, or $22^\circ\ 36'\ 55''$; side c, 281.785 yd., or 116.99 yd.

2. Given two sides of a triangle equal to 9459.31 feet and 8032.28 feet, and the angle opposite the first side equal to $71^\circ\ 3'\ 34''$; to find the other parts.

Solution.

$a = 9459{,}31$	ar. co. log 6.024141	log 3.975859
$b = 8032.28$	log 3.904839	
$A = 71^\circ\ 3'\ 34''$	log sin 9.975824	ar. co. log sin 0.024176
$B = 53^\circ\ 26'$	log sin 9.904804	
$C = 180^\circ - 124^\circ\ 29'\ 34'' = 55^\circ\ 30'\ 26''$		log sin 9.916032
	$c = 8242.64$	log 3.916067

Ans. Side, 8242.64 feet; angles, $53^\circ\ 26'$, $55^\circ\ 30'\ 26''$.

3. Given two sides of a triangle equal to 80 rods and 142.6

rods, and the angle opposite the second side equal to 96°, to solve the triangle.

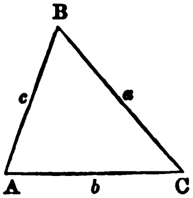
4. Given in a triangle ABC, the side a equal to 32.1098 rods, the side b equal to 125.701 rods, and the angle A equal to 14° 48′; to solve the triangle.

Ans. Angle B, 90°; angle C, 75° 12′; side c, 121.531 rods.

5. Given two sides of a triangle equal to 1540.37 feet and 760.9 feet, and the angle opposite the second equal to 30° 22′ 8″; to find the other side and angles. Ans. Impossible.

CASE III.

128. *Given two sides and the included angle.*



Let there be given in the triangle ABC the sides a and b and the included angle C, to solve the triangle.

To find A and B, we have

$$A + B = 180° - C,$$

and

$$\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2}C = \text{complement of } \tfrac{1}{2}C;$$

whence,

$$\tan\tfrac{1}{2}(A + B) = \cot\tfrac{1}{2}C. \qquad (131)$$

Then, the half difference of A and B is found by means of (95), which gives

$$a + b : a - b :: \tan\tfrac{1}{2}(A + B) : \tan\tfrac{1}{2}(A - B);$$

whence,

$$\tan\tfrac{1}{2}(A - B) = \frac{a-b}{a+b}\tan\tfrac{1}{2}(A + B) = \frac{a-b}{a+b}\cot\tfrac{1}{2}C, \quad (132)$$

or, by logarithms,

$$\log\tan\tfrac{1}{2}(A-B) = \log(a-b) - \log(a+b) + \log\tan\tfrac{1}{2}(A+B)$$
$$= \log(a - b) - \log(a + b) + \log\cot\tfrac{1}{2}C. \qquad (133)$$

That is,

The logarithmic tangent of half the difference of the two required angles is equal to the logarithm of the difference of the *given sides, minus the logarithm of their sum, plus the logarithmic tangent of half the sum of the required angles, or plus the logarithmic cotangent of half the given angle.*

Since $\frac{1}{2}(A+B)$ is known, when $\frac{1}{2}(A-B)$ is found, we have

$$A = \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B),$$

$$B = \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A-B).$$

That is,

The GREATER *of the two required angles is equal to half their sum, plus half their difference; and the* SMALLER *angle is equal to half their sum, minus half their difference.*

To find c. By (128), we have

$$\log c = \log a + \log \sin C - \log \sin A.$$

EXAMPLES.

1. Given of any triangle ABC, the side a equal to 9459.31 feet, the side b equal to 8032.28 feet, and the included angle C equal to 55° 30 26″; to find the side c and the angles A and B.

Solution. $A + B = 180° - C = 124° 29' 34''$, and $\frac{1}{2}(A+B) = 62° 14' 47''$. Then, by (133),

$a + b =$	17491.59	ar. co. log	5.757170
$a - b =$	1427.03	log	3.154433
$\frac{1}{2}(A+B) =$	62° 14′ 47″	log tan	10.278844
$\frac{1}{2}(A-B) =$	8° 48′ 47″	log tan	9.190447
$B =$	53° 26′		
$A =$	71° 3′ 34″	ar. co. log sin	0.024176
$C =$	55° 30′ 26″	log sin	9.916032
$a =$	9459.31	log	3.975859
$c =$	8242.64	log	3.916067

Ans. Side c, 8242.64 ft.; angle A, 71° 3′ 34″; angle B, 53° 26′.

2. Given two sides of a triangle equal to 142.6 feet and 110 feet, and the included angle equal to 33° 55′; to solve the triangle.

3. Given the two sides of a triangle equal to 153 rods and 137 rods, and the included angle equal to 40° 33′ 12″; to find the other parts.

Ans. Side, 101.615 feet; angles, 78° 13′ 1″ and 61° 13′ 47″.

Log tan ½ (A − B) = [illegible]

− [illegible]

+ [illegible] tan ½ (A + B)

s;

e

6.4 [illegible]

9.4 [illegible] 2

10.2 [illegible]

1.361728

8.911422

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(109), (110), and (111) for the [illegible]

When the half angle is less than 45°, the table will determine it from its sine with greater precision than from the cosine, and *vice versa* when the half angle is greater than 45°.

The method by the tangent of half the angle is precise, and requires the use of but four logarithms.

NOTE. This case may also be solved by drawing a perpendicular from the vertex to the base of the triangle, thus dividing it into two right-angled triangles, of which the hypothenuses are known, and the sum of whose bases is the base of the original triangle. Let s and s' represent CD and DA (Fig. Art. 113), then (Geom., Prop. XI. Bk. IV.),

$$p^2 = c^2 - s'^2 = a^2 - s^2, \quad \text{or,} \quad s^2 - s'^2 = a^2 - c^2,$$

whence, $$(s + s')(s - s') = (a + c)(a - c).$$

Substituting b for $s + s'$, $$s - s' = \frac{(a+c)(a-c)}{b},$$

a form to which logarithms can be readily applied.

Knowing $s + s'$ and $s - s'$, s and s' can at once be found, and thence the angles A, C, and B, by Art. 122.

EXAMPLES.

1. Given of any triangle ABC, the side a equal to 216 yards, the side b equal to 217 yards, and the side c equal to 235 yards; to find the angles A, B, and C.

Solution. By (134), (135), and (136) we have

$a = 216$		ar. co. log 7.665546	ar. co. log 7.665546
$b = 217$	ar. co. log 7.663540		ar. co. log 7.663540
$c = 235$	ar. co. log 7.628932	ar. co. log 7.628932	
$s - a = 118$		log 2.071882	log 2.071882
$s - b = 117$	log 2.068186		log 2.068186
$s - c = 99$	log 1.995635	log 1.995635	
	2) 19.356293	2) 19.361995	2) 19.469154
log sines	9.678147	9.680998	9.734577

$\frac{1}{2} A = 28° 27' 47''$; $\frac{1}{2} B = 28° 40' 4''.4$; $\frac{1}{2} C = 32° 52' 8''.6$.

Ans. $A = 56° 55' 34''$; $B = 57° 20' 8''.8$; $C = 65° 44' 17''.2$.

2. Given the three sides of a triangle equal to 432, 543, and 654; to solve the triangle by means of the cosine.

3. Given the three sides of a triangle equal to 95.12, 162.34, and 98; to solve the triangle by means of the tangent.

Ans. The angles, 32° 14′ 53″; 114° 24′ 9″; 33° 20′ 58″.

BOOK IV.

PRACTICAL APPLICATIONS.

DETERMINATION OF HEIGHTS AND DISTANCES.

131. A HORIZONTAL PLANE is one which is parallel to the horizon.

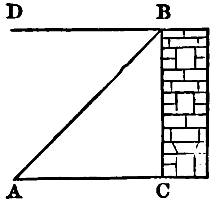
A VERTICAL PLANE is one which is perpendicular to a horizontal plane.

A HORIZONTAL LINE is one which is parallel to the horizon.

A VERTICAL LINE is one which is perpendicular to a horizontal plane.

132. A HORIZONTAL ANGLE is one the plane of whose sides is horizontal.

A VERTICAL ANGLE is one the plane of whose sides is vertical.

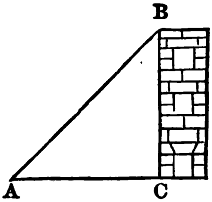


An ANGLE OF ELEVATION is a vertical angle having one side horizontal and the inclined side above it; as the angle CAB.

An ANGLE OF DEPRESSION is a vertical angle having one side horizontal and the inclined side under it; as the angle DBA.

133. *To determine the height of a vertical object standing on a horizontal plane.*

Let B be the top of the object, and let it be required to find its height BC.



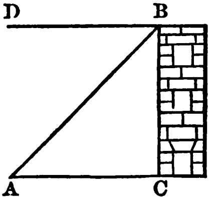
Measure from the foot of the object, in the horizontal plane, any convenient distance, as *A C*, as a base line, and at *A* observe the angle of elevation *C A B*. Then, in the right-angled triangle *A B C*, we have known the side *A C* and the acute angle *A*; therefore we can determine the height *B C* by Art. 121.

EXAMPLES.

1. Standing on the edge of a moat 40 feet wide, I observe that the wall of a fort upon the opposite brink subtends an angle at the point of observation of 36° 52′ 12″; required the height of the wall. Ans. 30 feet.

2. The angle of elevation of the top of a flag-staff, measured on a horizontal plane, at a distance of 89 feet from the foot of the staff, is 41° 29′; what is the height of the staff?

134. *To find the distance of a vertical object, its height being given.*



Let *B C* be the object whose height is given, and let it be required to find the distance *A C*.

Measure the angle of elevation *C A B*, or the angle of depression *D B A*, which is equal to *C A B*. Then, in the right-angled triangle *A B C*, we have known the side *B C* and the angles; therefore we can find the distance *A C* by Art. 121.

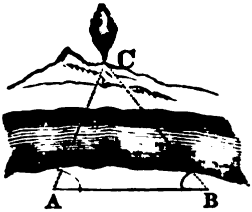
EXAMPLES.

1. A tree 91 feet in height stands on the same horizontal plane with a dial, at which the angle of elevation subtended by the tree is 32° 22′; required the distance of the dial from the foot of the tree. Ans. 143.6 feet.

2. From the top of a house whose height is 30 feet, I observe that the angle of depression of an object standing on the same horizontal plane with the house is 36° 52′ 12″; required the

distance of the object from the base of the house, and also the length of the line that will just connect the object with the top of the house.

135. *To find the distance of an inaccessible point on a horizontal plane.*



Let C be the point inaccessible from A and B, and let it be required to find its distance from each of those points.

Measure as a horizontal base line the distance between A and B, and observe the horizontal angles CAB and CBA. Then, in the triangle ABC, there will be known the side AB and the angles; therefore the sides AC and BC can be found by Art. 125.

EXAMPLES.

1. Wanting to know the distances of two objects from a tree, inaccessible by reason of an intervening river, I measured the distance in a straight line between the two objects, and found it to be 772.45 feet; I also found the horizontal angles formed by the extremities of the straight line with the tree to be 80° 58′ 4″ and 43° 33′ 44″. Required the distances of the objects from the tree. Ans. The one, 926.01 feet; the other, 646.16 feet.

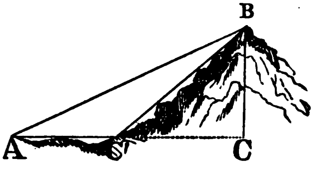
2. Two ships are engaged in cannonading a fort by the seaside; the ships are 131.89 rods apart, and the two angles at the ends of the straight line connecting the ships, formed by that line and lines drawn to the fort, are 18° 52′ 13″ and 152° 11′ 42″. Required the distance of each ship from the fort.

136. *To find the height of an inaccessible object above a horizontal plane.*

First Method. Let B be the top of the object, and let it be required to find the height BC.

Measure a horizontal base line, AC', of any convenient length, directly toward the object, and observe the angles of elevation at A and C'. Then, in the triangle ABC', since

$BC'A$ is the supplement of $CC'B$, we have known the side AC' and all the angles; therefore we can find the side AB by Art. 125. Then, in the right-angled triangle ABC, we have known the hypothenuse AB and the angles; therefore we can find the height BC by Art. 120.



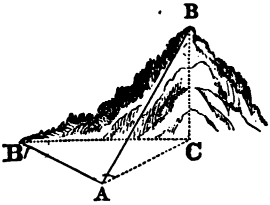
EXAMPLES.

1. Required the altitude of a hill whose angle of elevation, taken at the foot of it, was 55° 54′, and 300 feet back, on the same horizontal plane with the foot, the angle was 33° 20′.

Ans. 355.71 feet.

2. Two observers at sea, 800 yards apart, noticed at the same instant a meteor bearing due east from each; to the one its angle of elevation was 57°, and to the other the same angle was 31° 28′. Required the altitude of the meteor above the horizontal plane of the ships.

Second Method. Let B be the top of the object, and let it be required to find the height BC. Now, suppose it is not convenient to measure a horizontal base line directly toward the object, and we measure it in *any* direction, AB', also measuring the angles CAB' and $CB'A$. Then, in the horizontal triangle $AB'C$, we know the side AB' and all the angles; therefore the side AC can be found by Art. 125. Then, also, by observing the angle of elevation CAB, we shall, in the right-angled triangle ABC, know the side AC and all the angles; therefore the height BC can be found by Art. 121.



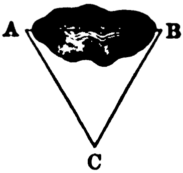
EXAMPLE.

1. A person on one side of a river observed an eagle's nest on an inaccessible mountain-crag on the opposite side, and being desirous of ascertaining its height above the level of the river, he measured along the shore a straight line 110 yards in length, and

found the horizontal angles of its extremities with the object to be 33° 55′ and 96°, and also the angle of elevation at the latter to be 45°. Required the height of the nest above the water.

Ans. 240 feet.

137. *To find the distance between two objects separated by an impassable barrier.*



Let A and B be two objects separated by an impassable barrier, and let it be required to find the distance, $A\,B$, between them.

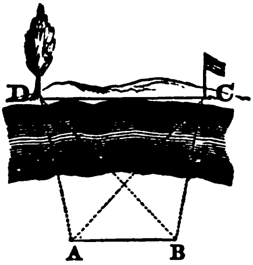
Take any point, C, from which A and B are both visible and accessible. Measure $C\,A$ and $C\,B$, and also note the angle $A\,C\,B$. Then, since in the triangle $A\,B\,C$ the two sides $C\,A$ and $C\,B$, with their included angle, are known, the distance $A\,B$ can be found by Art. 128.

EXAMPLES.

1. Two bounds of a lot have between them an impassable morass, and, wishing to find their distance apart, I have taken their distances from a third point, which could be seen from each. These distances are 124.75 and 171.41 rods, and the angle at that point subtended by the bounds is 99° 25′. How far are the bounds apart? Ans. 227.91 rods.

2. The distance between two trees cannot be directly measured, in consequence of an intervening obstacle, but within sight of each is a third tree, and their distances from this are known to be 274.65 and 396.11 yards, and the angle at that point subtended by the two trees is 8° 56′ 5″. Required the distance between the two trees.

138. *To find the distance between two inaccessible objects.*

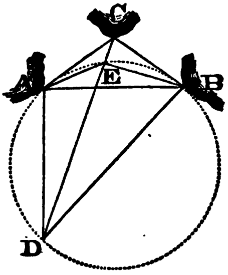


Let C and D be the objects, and A and B two accessible points, from which both the objects are visible. Measure the base line $A\,B$, and observe the angles $D\,A\,B$, $D\,B\,A$, $C\,A\,B$, and $C\,B\,A$. Then, in the triangle DAB, since we have the side AB and all the angles, we can find the side BD by Art. 125. In the triangle ABC we have the side AB and all the angles, hence we can find BC. Then, BD and BC being found, we have in the triangle BCD the sides BD and BC, with their included angle; therefore we can find the distance CD by Art. 128.

EXAMPLE.

1. Wanting to ascertain the distance between a tree, D, and a flagstaff, C, on the opposite side of a river from me, I measured along the shore, on the horizontal plane with the objects, a base line, AB, of 110 yards. At A, the angle DAB equals 96°, and CAB equals 29° 56′; at B, the angle DBA equals 33° 55′, and CBA equals 133° 50′. Required the distance between the tree and the flagstaff. Ans. 261.81 yards.

139. *To find the distances from a given point, of three objects whose distances from each other are known.*



Let it be required to find the distances from D, a given point, of three objects, A, B, and C, whose distances from each other are known.

Observe the angles ADC and BDC. Describe a circle about the triangle ADB, and draw AE and EB; then the angle ABE is equal to the angle ADE, since both are measured by half of the same arc AE (Geom., Prop. XVIII. Bk. III.); also the angle BAE is equal to the angle BDE, for a like reason. Now, in the triangle AEB, the side AB and all the angles are known, hence the side AE may be found by Art. 125. Again, the sides of the triangle ABC being given, we may find the angle BAC by Art. 129; then, in the triangle AEC, there will be known the two sides AC, AE, and the included angle CAE, so that the angle ACE may be found by Art. 128. Then, in the triangle ACD, we shall know the side AC and the angles ACD and ADC; therefore we can find the distance AD

by Art. 125, and thence the other two distances, $C\,D$ and $B\,D$.

EXAMPLE.

1. On approaching a harbor, at the point D, I observed three headlands, A, B, and C. Now it appeared from a chart that the distance from A to B was 800 yards, from A to C 600 yards, and from B to C 400 yards; the angle $A\,D\,C$ I found by observation to be 33° 45′, and the angle $B\,D\,C$ to be 22° 30′. What was the distance of each of the headlands from me?

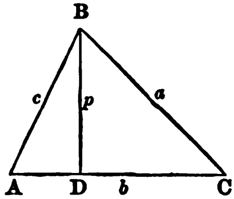
Ans. A, 710.19 yards; B, 934.29 yards; C, 1042.52 yards.

DETERMINATION OF AREAS.

140. The **Area** of any figure, or its quantity of surface, is determined by the number of times the given surface contains some other area, assumed as the unit of measure, as a square *inch*, a square *foot*, &c.

The areas of *parallelograms*, *triangles*, *trapezoids*, &c. can be determined by direct application of the principles of Geometry; but sometimes it is convenient to determine areas, especially of triangles, by means of their lines and angles, which requires the aid of Trigonometry.

141. *To find the area of a triangle by means of two of its sides and the included angle.*



Let $A\,B\,C$ be any plane triangle, in which are given the sides b and c and the included angle A, to find the area of the triangle. Draw the perpendicular, p, from B to the opposite side, $A\,C$; then, since the area of the triangle is equal to half the product of its base by its altitude (Geom., Prop VI. Bk. IV.),

$$\text{area of } A\,B\,C = \tfrac{1}{2}\,b\,p. \tag{137}$$

But, by (85), $$p = c \sin A;$$

whence, $$\text{area } A\,B\,C = \tfrac{1}{2}\,b\,c \sin A, \tag{138}$$

or, by logarithms,

$$\log \text{area } A\ B\ C = \log \tfrac{1}{2}\, b + \log c + \log \sin A. \qquad (139)$$

That is,

The logarithm of the area of a triangle is equal to the logarithm of half of either side, plus the logarithm of either of the other sides, plus the logarithmic sine of their included angle.

EXAMPLE.

1. Required the area of a triangle which has two of its sides equal to 105 and 85 feet, and the included angle equal to 28° 5′.

Ans. 2100 sq. ft.

142. In like manner, the area of any *parallelogram* may be found, when two of its adjacent sides and the included angle are known; for the diagonal divides a parallelogram into two equal triangles (Geom., Prop. XXXI. Cor. 1, Bk. I.).

EXAMPLE.

1. What is the area of a piece of ground, in the form of a parallelogram, which has two adjacent sides equal, respectively, to 120 and 212 rods, and their included angle equal to 85° 30′?

143. *To find the area of a triangle by means of a side and the angles.*

In the triangle $A\ B\ C$ (Fig. Art. 141), let the side c and the angles be given, to find the area of the triangle. By means of Art. 111 we have

$$b = \frac{c \sin B}{\sin C}; \qquad (140)$$

and by (85)

$$p = c \sin A.$$

Substituting these values in (137), we obtain

$$\text{area of } A\ B\ C = \frac{c^2 \sin A\ \sin B}{2 \sin C}, \qquad (141)$$

or, by logarithms,

$$\log 2 \text{ area } ABC = 2 \log c + \log \sin A + \log \sin B - \log \sin C. \quad (142)$$

That is,

The logarithm of double the area of a triangle is equal to twice the logarithm of either side, plus the logarithmic sines of its adjacent angles, minus the logarithmic sine of its opposite angle.

EXAMPLES.

1. A triangular lot has a side equal to 45 rods, and the adjacent angles equal to 70° and 69° 40′; required the area of the lot. Ans. 1378.41 sq. rods.

2. Given of a triangular field $A\,B\,C$, the angle A equal to 31° 27′, the angle B equal to 101° 31′, and the included side $A\,B$ equal to 30 rods; required the area of the field.

144. *To find the area of a triangle by means of its three sides.*

Let $A\,B\,C$ (Fig. Art. 141) be the given triangle. Then, by (138),

$$\text{area of } A\,B\,C = \tfrac{1}{2}\,b\,c \sin A;$$

but, by taking twice the product of the values of $\sin \frac{1}{2} A$ and $\cos \frac{1}{2} A$, in (102) and (106), we have, by (74),

$$\sin A = \frac{2}{b\,c}\sqrt{s\,(s-a)\;(s-b)\;(s-c)}, \qquad (143)$$

in which s denotes half the sum of the sides of the triangle.

Substituting in the preceding equation this value of $\sin A$, we obtain

$$\text{area of } A\,B\,C = \tfrac{1}{2}\,b\,c \times \frac{2}{b\,c}\sqrt{s\,(s-a)\;(s-b)\;(s-c)};$$

whence, $$\text{area of } A\,B\,C = \sqrt{s\,(s-a)\;(s-b)\;(s-c)}; \qquad (144)$$

or, by logarithms,

$$\log \text{area } ABC = \frac{\log s + \log(s-a) + \log(s-b) + \log(s-c)}{2}. \qquad (145)$$

That is,

The logarithm of the area of a triangle is equal to half the sum of the logarithm of half the sum of the sides and the logarithms of the remainders obtained by taking each side separately from half the sum of the sides.

EXAMPLES.

1. Given the three sides of a triangle equal to 30, 40, and 60 rods, respectively; required the area of the triangle.

Ans. 533.27 sq. rods.

2. A certain fort is in the form of an equilateral triangle, whose sides are each 600 feet; required the area occupied by the fort.

MISCELLANEOUS PROBLEMS.

1. The angle of elevation of a vertical tower is observed to be 30°, at the end of a horizontal base line of 100 yards, measured from its foot. Required the height of the tower.

2. A rope-dancer wishes to ascend a steeple 100 feet high, by means of a rope 196 feet long. If he can do so, find at what inclination he must be able to walk up the rope.

3. From the summit of a pier which rises 100 feet above the margin of a river, the angle of depression of the opposite margin was found to be 33° 16′. Required the width of the river.

4. If the distance of the moon from the earth be taken at 238500 miles, and the angle subtended by the semidiameter of the moon be 15′ 33″.5 at that distance, what is the moon's diameter? Ans. 2158 miles.

5. A point of land was observed by a ship at sea to bear east by south, that is, 11° 15′ S. of E.; and after sailing northeast 12 miles, it was found to bear southeast by east, that is, 33° 45′ S. of E. Required the distance of the headland from the ship at the last observation. Ans. 26.07 miles.

6. From the top of Mont Blanc, 3 miles high, the angle of depression of the remotest visible point of the earth's surface is 2° 13′ 27″. Required the diameter of the earth, supposing it to be a perfect sphere; and, also, the utmost distance from which the mountain is visible.

Ans. Diameter, 7958 miles; distance, 154.5 miles.

7. A side of the base of a square pyramid is 200 feet, and each edge is 150 feet; required the slope of each face.

Ans. 26° 34′, nearly.

8. I have a meadow in the form of a parallelogram, whose two adjacent sides are 20 rods and 18 rods, including an angle of 78° 9′; the same has been divided into two equal lots by a fence running diagonally. Required the area of each lot.

Ans. 176.16 square rods.

9. A traveler wishing to know the distance and height of a mountain-top over which he had to pass, took the angle of its

elevation at two stations, in a direct line towards it, the one 3 miles, or 5280 yards, nearer the mountain than the other, and found the angles to be 2° 45′ and 3° 20′. Required the horizontal distance of the mountain-top from the nearer station, and its height. Ans. Distance, 24840 yards; height, 1447 yards.

10. From the top of a light-house the angle of depression of a ship at anchor was observed to be 4° 52′, from the bottom of the light-house the angle was 4° 2′. Required the horizontal distance of the vessel, and the height of the hill on which the light-house is placed, the height of the light-house being 60 feet.

Ans. Horizontal distance, 4100.4 feet; height, 289.12 feet.

11. When a tower 150 feet high throws a shadow 75 feet long upon the horizontal plane on which the tower stands, what is the sun's altitude (Art. 189)? Ans. 63° 26′ 6″.

12. The sides of a triangle are equal to 3 and 12, respectively, and the included angle is 30°; find the hypothenuse of an equal right-angled isosceles triangle. Ans. 6.

13. From a window near the bottom of a house, which seemed to be on a level with the bottom of a steeple, I took the angle of elevation of the top of the steeple, equal to 40°; then from another window, 18 feet directly above the former, the like angle was 37° 30′. Required the height and distance of the steeple.

Ans. Height, 210.4 feet; distance, 250.8 feet.

14. Two pulleys, whose diameters are 6 inches and 4 feet 3 inches, respectively, are placed at a distance of 3 feet 6 inches from centre to centre. What must be the length of a belt which shall connect them, by passing around their circumferences, without crossing? Ans. 15 feet 5.9 inches.

15. A tower is surrounded by a circular moat. At noon on a certain day, the shadow of the top of the flag-staff is observed to project 45 feet beyond the edge of the moat. When the sun is due west, on the same day, the shadow projects 120 feet beyond the moat. The distance between the extremities of the shadows is 375 feet. The angle of elevation of the top of the flag-staff from any point of the edge of the moat is 60°. Find the height of the tower, and the altitude of the sun at noon.

Ans. 311.77 feet; 54° 10′ 57″.

BOOK V.

SPHERICAL TRIGONOMETRY.

DEFINITIONS.

145. SPHERICAL TRIGONOMETRY treats of methods of computing spherical triangles.

146. A SPHERICAL TRIANGLE is a portion of the surface of a sphere bounded by three arcs of a great circle, each of which is less than a semi-circumference.

The three planes in which the arcs lie form a polyedral angle at the centre of the sphere.

The ANGLES of a spherical triangle are the diedral angles made by the plane faces which form the polyedral angle.

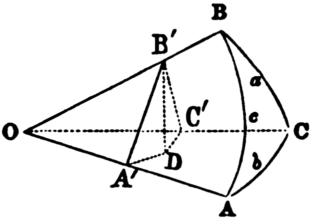
147. The sides and angles of spherical triangles are usually both expressed in degrees, minutes, &c.

The circumference, however, is sometimes supposed to be divided into 24 equal parts, called *hours;* each hour into 60 equal parts, called *minutes of time;* each minute into 60 equal parts, called *seconds of time.* Then a side is expressed by the number of hours, minutes, seconds, and decimal parts of a second, which it contains.

Hours, minutes, and seconds are denoted by *h.*, *m.*, and *s.* Thus, 3h. 35m. 5.8s.

RELATIONS BETWEEN THE SIDES AND ANGLES OF SPHERICAL TRIANGLES.

148. *In any spherical triangle, the sines of the sides are proportional to the sines of the opposite angles.*



Let ABC be any spherical triangle; A, B, and C the angles opposite to its sides a, b, and c, respectively; and O the centre of the sphere.

Take any point B' in OB, and draw $B'D$ perpendicular to the plane AOC; from D draw DA', DC', perpendicular to OA, OC, respectively; join $B'A'$, $B'C'$.

$B'C'O$ is a right angle (Geom., Prop. VI. Bk. VII.); therefore,

$$B'C' = OB' \sin B'OC' = OB' \sin a,$$

and

$$B'D = B'C' \sin B'C'D = B'C' \sin C = OB' \sin a \sin C.$$

In like manner,

$$B'D = OB' \sin c \sin A;$$

and, by the two preceding equations,

$$OB' \sin a \sin C = OB' \sin c \sin A,$$

whence,
$$\frac{\sin a}{\sin c} = \frac{\sin A}{\sin C}; \qquad (146)$$

or, in the form of a proportion,

$$\sin a : \sin c :: \sin A : \sin C. \qquad (147)$$

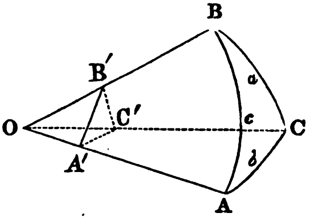
In a similar way it may be proved that

$$\sin a : \sin b :: \sin A : \sin B, \qquad (148)$$

$$\sin c : \sin b :: \sin C : \sin B. \qquad (149)$$

The figure supposes a, c, B, C, &c. each less than 90°, but the relation stated may be shown to hold when the figure is modified to meet any case whatever. For instance, if C alone is greater than 90°, the point D will fall beyond OC, instead of between OC and OA; then, $B'C'D$ will be the supplement of C, and thus, since the sine of an angle and its supplement are the same, the sine of $B'C'D$ is still equal to the sine of C.

149. *In any spherical triangle, the cosine of any side is equal to the product of the cosines of the other two sides, plus the product of the sines of those two sides into the cosine of their included angle.*



Let ABC be any spherical triangle, O the centre of the sphere.

Draw the plane $B'A'C'$ perpendicular to OA. Then the angle $B'A'C'$ is equal to the angle A, the angle $B'OC'$ measures the side a, and in the triangles $A'B'C'$, $OB'C'$ we have, by Art. 113,

$$\overline{B'C'}^2 = \overline{A'B'}^2 + \overline{A'C'}^2 - 2\,A'B' \times A'C' \cos A,$$

$$\overline{B'C'}^2 = \overline{OB'}^2 + \overline{OC'}^2 - 2\,OB' \times OC' \cos a.$$

Subtracting the first equation from the second, observing that $\overline{OB'}^2 - \overline{A'B'}^2$ and $\overline{OC'}^2 - \overline{A'C'}^2$ are each equal to $\overline{OA'}^2$, since the triangles $OA'B'$, $OA'C'$ are right-angled at A', we have

$$0 = 2\,\overline{OA}^2 + 2\,A'B' \times A'C' \cos A - 2\,OB' \times OC' \cos a;$$

$$\text{therefore,} \quad \cos a = \frac{OA' \times OA'}{OB' \times OC'} + \frac{A'B' \times A'C'}{OB' \times OC'} \cos A.$$

Substituting the functions derived from the triangles $OA'B'$, $OA'C'$, we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \qquad (150)$$

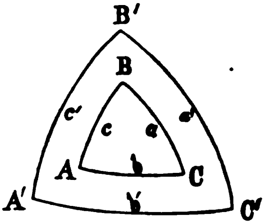
In like manner may be deduced

$$\cos b = \cos c \cos a + \sin c \sin a \cos B, \qquad (151)$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \qquad (152)$$

The preceding construction supposes the sides b and c, which contain the angle A, to be both less than 90°, but the formulæ obtained may be shown to be applicable in all cases.

150. *In any spherical triangle, the cosine of any angle is equal to the product of the sines of the other two angles into the cosine of their included side, minus the product of the cosines of those two angles.*



Let $A'B'C'$ be the polar triangle of $A\ B\ C$; denote its angles by A', B', and C', and its sides by a', b', and c'. Then (Geom., Prop. IX. Bk. IX.),

$$A' = 180^\circ - a,\quad B' = 180^\circ - b,\quad C' = 180^\circ - c;$$

$$a' = 180^\circ - A,\quad b' = 180^\circ - B,\quad c' = 180^\circ - C.$$

Applying (150) to $A'B'C'$, we have

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A';$$

or, by (46), $-\cos A = \cos B \cos C - \sin B \sin C \cos a$;

whence, $$\cos A = \sin B \sin C \cos a - \cos B \cos C. \qquad (153)$$

In like manner may be deduced

$$\cos B = \sin C \sin A \cos b - \cos C \cos A, \qquad (154)$$

$$\cos C = \sin A \sin B \cos c - \cos A \cos B. \qquad (155)$$

151. *In any spherical triangle, the cotangent of one side into the sine of another side is equal to the cotangent of the angle opposite the first side into the sine of the included angle, plus the cosine of the second side into the cosine of the included angle.*

By (150) and (152) we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C;$$

and by means of (147),

$$\sin c = \sin a \frac{\sin C}{\sin A}.$$

Substituting these values of $\cos c$ and $\sin c$ in the first equation, we obtain

$$\cos a = (\cos a \cos b + \sin a \sin b \cos C) \cos b + \frac{\sin a \sin b \cos A \sin C}{\sin A};$$

or,

$$\cos a = \cos a \cos^2 b + \sin a \sin b \cos b \cos C + \sin a \sin b \cot A \sin C.$$

Therefore, transposing $\cos a \cos^2 b$, and observing that, by (11),

$$\cos a - \cos a \cos^2 b = \cos a \sin^2 b,$$

we have

$$\cos a \sin^2 b = \sin a \sin b \cot A \sin C + \sin a \sin b \cos b \cos C,$$

and dividing the whole by $\sin a \sin b$, we obtain

$$\cot a \sin b = \cot A \sin C + \cos b \cos C. \qquad (156)$$

152. By interchanging the letters in (156), we obtain

$$\cot a \sin c = \cot A \sin B + \cos c \cos B, \qquad (157)$$

$$\cot b \sin a = \cot B \sin C + \cos a \cos C, \qquad (158)$$

$$\cot b \sin c = \cot B \sin A + \cos c \cos A, \qquad (159)$$

$$\cot c \sin a = \cot C \sin B + \cos a \cos B, \qquad (160)$$

$$\cot c \sin b = \cot C \sin A + \cos b \cos A. \qquad (161)$$

153. The formulæ developed in the preceding articles are general, and apply to every case of spherical triangles, but require some transformations to render them more convenient for logarithmic computations.

The formulæ (150), (151), and (152) of Art. 149 are considered the *fundamental formulæ* of spherical trigonometry, since from them all its other formulæ may be deduced.

154. *To express the sine, cosine, and tangent of half an angle of a triangle as functions of the sides.*

By means of (150) we have

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}, \qquad (162)$$

but this formula is not suited to logarithmic computation.

We then subtract each member of the equation from 1, and obtain (Art. 63),

$$1 - \cos A = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c} = \frac{\cos (b - c) - \cos a}{\sin b \sin c}.$$

Substituting for $1 - \cos A$ its value, $2 \sin^2 \frac{1}{2} A$ (80), we obtain

$$2 \sin^2 \tfrac{1}{2} A = \frac{\cos (b - c) - \cos a}{\sin b \sin c}.$$

Now if, in (65), we make $A = a$, and $B = b - c$,

$\frac{1}{2}(A + B) = \frac{1}{2}(a + b - c)$, $\frac{1}{2}(A - B) = \frac{1}{2}(a - b + c)$,

then,

$\cos (b - c) - \cos a = 2 \sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)$,

which, substituted in the preceding equation, gives

$$\sin^2 \tfrac{1}{2} A = \frac{\sin \frac{1}{2}(a - b + c) \sin \frac{1}{2}(a + b - c)}{\sin b \sin c}. \qquad (163)$$

Let, now, $s =$ half the sum of the sides of the triangle; then,

$$a + b - c = 2(s - c), \quad a - b + c = 2(s - b).$$

Substituting these values in the last equation, and reducing, we have

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}}. \qquad (164)$$

Similarly, $$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s - c) \sin (s - a)}{\sin c \sin a}}, \qquad (165)$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s - a) \sin (s - b)}{\sin a \sin b}}. \qquad (166)$$

Adding each member of equation (162) to 1, and observing that $1 + \cos A = 2 \cos^2 \frac{1}{2} A$ (81), by means of (65), we have

$$\cos^2 \tfrac{1}{2} A = \frac{\sin \frac{1}{2}(a + b + c) \sin \frac{1}{2}(b + c - a)}{\sin b \sin c}.$$

Introducing $s = \frac{1}{2}(a + b + c)$, and reducing, we have

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}}. \qquad (167)$$

Similarly, $$\cos \tfrac{1}{2} B = \sqrt{\frac{\sin s \sin (s - b)}{\sin c \sin a}}, \qquad (168)$$

$$\cos \tfrac{1}{2} C = \sqrt{\frac{\sin s \sin (s - c)}{\sin a \sin b}}. \qquad (169)$$

Again, dividing (164), (165), and (166) by (167), (168), and (169), respectively, we obtain

$$\tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}, \tag{170}$$

$$\tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\sin(s-b)}}, \tag{171}$$

$$\tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}. \tag{172}$$

155. *To express the sine, cosine, and tangent of half a side of a triangle as functions of the angles.*

By means of (153) we have

$$\cos a = \frac{\cos A + \cos B\cos C}{\sin B\sin C}. \tag{173}$$

Whence,

$$1 - \cos a = 1 - \frac{\cos A + \cos B\cos C}{\sin B\sin C} = -\frac{\cos A + \cos(B + C)}{\sin B\sin C},$$

and

$$\sin^2\tfrac{1}{2}a = -\frac{\cos\frac{1}{2}(A + B + C)\cos\frac{1}{2}(B + C - A)}{\sin B\sin C}. \tag{174}$$

Making $\frac{1}{2}(A + B + C) = S$, substituting, and reducing, we have

$$\left.\begin{aligned} \sin\tfrac{1}{2}a &= \sqrt{\frac{-\cos S\cos(S-A)}{\sin B\sin C}}. \\ \text{Also,}\qquad \sin\tfrac{1}{2}b &= \sqrt{\frac{-\cos S\cos(S-B)}{\sin C\sin A}}, \\ \sin\tfrac{1}{2}c &= \sqrt{\frac{-\cos S\cos(S-C)}{\sin A\sin B}}. \end{aligned}\right\} \tag{175}$$

In like manner, we obtain

$$\left.\begin{aligned} \cos\tfrac{1}{2}a &= \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B\sin C}}, \\ \cos\tfrac{1}{2}b &= \sqrt{\frac{\cos(S-C)\cos(S-A)}{\sin C\sin A}}, \\ \cos\tfrac{1}{2}c &= \sqrt{\frac{\cos(S-A)\cos(S-B)}{\sin A\sin B}}. \end{aligned}\right\} \tag{176}$$

Hence,

$$\left.\begin{aligned}\tan \tfrac{1}{2} a &= \sqrt{\frac{-\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}},\\ \tan \tfrac{1}{2} b &= \sqrt{\frac{-\cos S \cos (S-B)}{\cos (S-C) \cos (S-A)}},\\ \tan \tfrac{1}{2} c &= \sqrt{\frac{-\cos S \cos (S-C)}{\cos (S-A) \cos (S-B)}}.\end{aligned}\right\} \quad (177)$$

Since S is always greater than 90° and less than 270° (Geom., Prop. X. Bk. IX.), cos S is always negative, and therefore $-\cos S$ in the numerators of the first and third of the above sets of formulæ is essentially positive.

156. *To prove Napier's Analogics.*

Let $$m = \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b};$$

then, $$\sin A = m \sin a, \qquad \sin B = m \sin b,$$

$$\sin A + \sin B = m (\sin a + \sin b), \quad (178)$$

$$\sin A - \sin B = m (\sin a - \sin b). \quad (179)$$

By (153) and (154) we have

$$\cos A + \cos B \cos C = \sin B \sin C \cos a = m \sin C \cos a \sin b,$$

$$\cos B + \cos A \cos C = \sin A \sin C \cos b = m \sin C \sin a \cos b.$$

Adding these equations, factoring, and reducing by (17),

$$(\cos A + \cos B)(1 + \cos C) = m \sin C \sin (a + b). \quad (180)$$

Dividing (178) by (180), and multiplying by sin C,

$$\frac{\sin A + \sin B}{\cos A + \cos B} \times \frac{\sin C}{1 + \cos C} = \frac{\sin a + \sin b}{\sin (a + b)}. \quad (181)$$

Now, by means of (62), (63), and (74), we obtain

$$\frac{\sin a + \sin b}{\sin (a + b)} = \frac{\cos \tfrac{1}{2} (a - b)}{\cos \tfrac{1}{2} (a + b)}, \quad (182)$$

and
$$\frac{\sin a - \sin b}{\sin (a+b)} = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)}. \qquad (183)$$

Substituting in (181) the value of each expression, from (68), (84), and (182),

$$\tan \tfrac{1}{2}(A+B) \times \tan \tfrac{1}{2} C = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)},$$

or,

$$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A+B)}. \qquad (184)$$

In like manner, from (179) and (180),

$$\frac{\sin A - \sin B}{\cos A + \cos B} \times \frac{\sin C}{1 + \cos C} = \frac{\sin a - \sin b}{\sin (a+b)},$$

whence, by (69), (84), and (183),

$$\tan \tfrac{1}{2}(A-B) \times \tan \tfrac{1}{2} C = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)},$$

or,

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)}. \qquad (185)$$

Formulæ (184) and (185) may be thus expressed:

$$\cos \tfrac{1}{2}(a+b) : \cos \tfrac{1}{2}(a-b) :: \cot \tfrac{1}{2} C : \tan \tfrac{1}{2}(A+B), \qquad (186)$$

$$\sin \tfrac{1}{2}(a+b) : \sin \tfrac{1}{2}(a-b) :: \cot \tfrac{1}{2} C : \tan \tfrac{1}{2}(A-B). \qquad (187)$$

That is,

The cosine of half the sum of two sides of a spherical triangle is to the cosine of half their difference as the cotangent of half the included angle is to the tangent of half the sum of the other two angles.

The sine of half the sum of two sides of a spherical triangle is to the sine of half their difference as the cotangent of half the included angle is to the tangent of half the difference of the other two angles.

By applying (186) and (187) to the polar triangle, Art. 150, we obtain

$$\cos \tfrac{1}{2}(A+B) : \cos \tfrac{1}{2}(A-B) :: \tan \tfrac{1}{2} c : \tan \tfrac{1}{2}(a+b), \quad (188)$$

$$\sin \tfrac{1}{2}(A+B) : \sin \tfrac{1}{2}(A-B) :: \tan \tfrac{1}{2} c : \tan \tfrac{1}{2}(a-b). \quad (189)$$

That is,

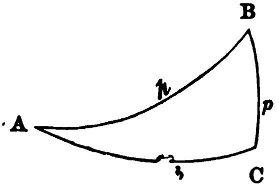
The cosine of half the sum of two angles of a spherical triangle is to the cosine of half their difference as the tangent of half the included side is to the tangent of half the sum of the other two sides.

The sine of half the sum of two angles of a spherical triangle is to the sine of half their difference as the tangent of half the included side is to the tangent of half the difference of the other two sides.

The above four proportions are called, from their inventor, *Napier's Analogies.*

RELATIONS BETWEEN THE SIDES AND ANGLES OF RIGHT-ANGLED SPHERICAL TRIANGLES.

157. *The sine of either oblique angle is equal to the sine of the opposite side, divided by the sine of the hypothenuse.*



Let $A\,B\,C$ be any spherical triangle, right-angled at C.

By means of (146) we have

$$\sin A = \frac{\sin p}{\sin h} \sin C;$$

but, as $C = 90°$, $\sin C = 1$, and

$$\sin A = \frac{\sin p}{\sin h}. \quad (190)$$

In like manner,

$$\sin B = \frac{\sin b}{\sin h}. \quad (191)$$

158. *The cosine of either oblique angle is equal to the tangent of the adjacent side, divided by the tangent of the hypothenuse.*

By means of (161) we have

$$\cot h \sin b = \cot C \sin A + \cos b \cos A\,\cdot$$

but, if $C = 90°$, then $\cot C = 0$, and

$$\cot h \sin b = \cos b \cos A,$$

or,
$$\cos A = \frac{\cot h \sin b}{\cos b} = \cot h \tan b\,;$$

whence,
$$\cos A = \frac{\tan b}{\tan h}. \tag{192}$$

Also, by means of (160),
$$\cos B = \frac{\tan p}{\tan h}. \tag{193}$$

159. *The tangent of either oblique angle is equal to the tangent of the opposite side, divided by the sine of the adjacent side.*

By means of (156) we have

$$\cot p \sin b = \cot A \sin C + \cos b \cos C;$$

but, by making $C = 90°$, $\sin C = 1$, $\cos C = 0$, and

$$\cot A = \cot p \sin b = \frac{\sin b}{\tan p},$$

or,
$$\tan A = \frac{\tan p}{\sin b}. \tag{194}$$

Also, by means of (158),
$$\tan B = \frac{\tan b}{\sin p}. \tag{195}$$

160. *The sine of either oblique angle is equal to the cosine of the other, divided by the cosine of its opposite side.*

By means of (154) we have

$$\cos B = \sin A \sin C \cos b - \cos A \cos C,$$

which, by making $C = 90°$, becomes

$$\cos B = \cos b \sin A,$$

whence,
$$\sin A = \frac{\cos B}{\cos b}. \tag{196}$$

In like manner, by means of (153),
$$\sin B = \frac{\cos A}{\cos p}. \tag{197}$$

161. *The cosine of the hypothenuse is equal to the product of the cosines of the other two sides.*

By means of (152) we have

$$\cos h = \cos p \cos b + \sin p \sin b \cos C,$$

which, by making $C = 90°$, becomes

$$\cos h = \cos p \cos b. \tag{198}$$

162. *The cosine of the hypothenuse is equal to the product of the cotangents of the two oblique angles.*

By means of (155) we have

$$\cos C = \sin A \sin B \cos h - \cos A \cos B,$$

which, by making $C = 90°$, becomes

$$\sin A \sin B \cos h = \cos A \cos B,$$

or,
$$\cos h = \frac{\cos A \cos B}{\sin A \sin B} = \cot A \cot B. \tag{199}$$

163. The preceding formulæ may readily be remembered from their similarity to the corresponding ones for plane triangles; and, for convenience of reference, they are brought together in the following

TABLE.

1.	$\sin A = \frac{\sin p}{\sin h}.$	2.	$\sin B = \frac{\sin b}{\sin h}.$
3.	$\cos A = \frac{\tan b}{\tan h}.$	4.	$\cos B = \frac{\tan p}{\tan h}.$
5.	$\tan A = \frac{\tan p}{\sin b}.$	6.	$\tan B = \frac{\tan b}{\sin p}.$
7.	$\sin A = \frac{\cos B}{\cos b}.$	8.	$\sin B = \frac{\cos A}{\cos p}.$
9.	$\cos h = \cos p \cos b.$	10.	$\cos h = \cot A \cot B.$

SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES.

164. The solution of spherical triangles is the process by which, when the values of a sufficient number of their six elements are given, we calculate the values of the remaining elements.

In order to solve a right-angled spherical triangle, two of its elements, other than the right angle, must be given.

165. The formulæ requisite for the solution of right-angled spherical triangles are readily furnished by means of the relations demonstrated in the foregoing articles. Thus,

$$\sin A = \frac{\sin p}{\sin h} \quad \text{gives} \quad \sin p = \sin A \sin h, \qquad (200)$$

$$\sin B = \frac{\sin b}{\sin h} \quad \text{“} \quad \sin b = \sin B \sin h, \qquad (201)$$

$$\cos A = \frac{\tan b}{\tan h} \quad \text{“} \quad \cos A = \cot h \tan b, \qquad (202)$$

$$\cos B = \frac{\tan p}{\tan h} \quad \text{“} \quad \cos B = \cot h \tan p, \qquad (203)$$

$$\tan B = \frac{\tan b}{\sin p} \quad \text{“} \quad \sin p = \cot B \tan b, \qquad (204)$$

$$\tan A = \frac{\tan p}{\sin b} \quad \text{“} \quad \sin b = \cot A \tan p, \qquad (205)$$

$$\sin B = \frac{\cos A}{\cos p} \quad \text{“} \quad \cos A = \sin B \cos p, \qquad (206)$$

$$\sin A = \frac{\cos B}{\cos b} \quad \text{“} \quad \cos B = \sin A \cos b, \qquad (207)$$

which, with equations (198) and (199),

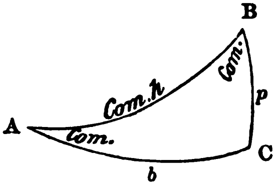
$$\cos h = \cos p \cos b, \qquad \cos h = \cot A \cot B,$$

enable us to determine every case of right-angled spherical triangles. For every one of these ten equations is a distinct combination, involving three of the five quantities, p, b, h, A, B; and five quantities, taken three at a time, can be combined only in ten different ways.

NAPIER'S CIRCULAR PARTS.

166. If, in any right-angled spherical triangle, the right angle be left out of consideration, the two sides adjacent to the right angle, and the complements of the hypothenuse and of the two other angles, are called the five *circular parts* of the triangle.

Thus, in the spherical triangle $A\ B\ C$, right-angled at C, the circular parts are p, b, and the complements of h, A, and B.



167. When any one of the five parts is taken for the *middle part*, the two adjacent to it, one on either side, are called the *adjacent parts*, and the other two parts are called the *opposite parts.* Then, whatever be the middle part, we have as

The Rules of Napier.

I. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

II. *The sine of the middle part is equal to the product of the cosines of the opposite parts.*

168. Napier's rules may be proved by showing that they agree with the results already established, Art. 165. Thus,

1. Let b be taken for the middle part; then p and the complement of A will be the adjacent parts, and the complements of B and h will be the opposite parts, and by the rules we have

$$\sin b = \tan (\text{com. } A) \tan p,$$

$$\sin b = \cos (\text{com. } B) \cos (\text{com. } h);$$

whence, by Art. 50,

$$\sin b = \cot A \ \tan p, \qquad \sin b = \sin B \sin h$$

which agree with (205) and (201).

In like manner, if p be taken as the middle part,

$$\sin p = \tan (\text{com. } B) \tan b,$$

$$\sin p = \cos (\text{com. } A) \cos (\text{com. } h);$$

whence,

$$\sin p = \cot B \ \tan b, \qquad \sin p = \sin A \ \sin h,$$

which agree with (204) and (200).

2. Let the complement of h be taken as the middle part;

then the complements of A and B will be the adjacent parts, p and b the opposite parts, and we have

$$\sin(\text{com. } h) = \tan(\text{com. } A)\ \tan(\text{com. } B),$$

$$\sin(\text{com. } h) = \cos p\ \cos b;$$

whence,

$$\cos h = \cot A\ \cot B, \qquad \cos h = \cos p\ \cos b,$$

which agree with (199) and (198).

3. Let the complement of A be taken as the middle part; then b and the complement of h will be the adjacent parts, p and the complement of B the opposite parts, and we have

$$\sin(\text{com. } A) = \tan(\text{com. } h)\ \tan b,$$

$$\sin(\text{com. } A) = \cos(\text{com. } B)\ \cos p;$$

whence,

$$\cos A = \cot h\ \tan b, \qquad \cos A = \sin B\ \cos p,$$

which agree with (202) and (206).

In like manner,

$$\sin(\text{com. } B) = \tan(\text{com. } h)\ \tan p,$$

$$\sin(\text{com. } B) = \cos(\text{com. } A)\ \cos b;$$

whence,

$$\cos B = \cot h\ \tan p, \qquad \cos B = \sin A\ \cos b,$$

which agree with (203) and (207).

169. Any element of a spherical triangle is less than 180° (Geom., Art. 505, 539). Two parts are said to be of the *same species* when they are in the same quadrant, that is, when they are both less, or both greater, than 90°; and of *different species* when one terminates in the first and the other in the second quadrant.

170. In order to determine whether a part sought is less or greater than 90°, the algebraic signs of the terms should be observed, according to Art. 68 or 78. When, however, the part sought is determined by its sine, since the sines in both the first and second quadrants are positive, there will be two solutions,

unless the ambiguity be removed by one of the following rules: —

1. *In any right-angled spherical triangle, an oblique angle and its opposite side are always of the same species.*

For, by (205), $\sin b = \cot A \ \tan p$,

in which, since $\sin b$ is always positive, $\cot A$ and $\tan p$ must always have the same sign, that is, A and p must be of the same species.

2. *When the two sides about the right angle are of the same species, the hypothenuse is less than 90°, but when they are of different species, the hypothenuse is greater than 90°.*

For, by (198), $\cos h = \cos p \ \cos b$,

in which, if $\cos p$ and $\cos b$ have the same signs, $\cos h$ will be positive, but if they have unlike signs, $\cos h$ will be negative.

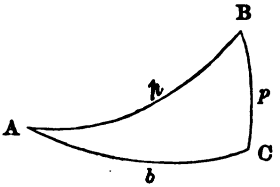
171. In the solution of right-angled spherical triangles, there will be six cases to consider, in which there may be given, respectively,

I. The hypothenuse and an oblique angle.
II. The hypothenuse and one side.
III. One side and its adjacent oblique angle.
IV. One side and its opposite oblique angle.
V. The two sides about the right angle.
VI. The two oblique angles.

CASE I.

172. *Given the hypothenuse and an oblique angle.*

Let there be given in the right-angled spherical triangle ABC, the hypothenuse h and the oblique angle A; to solve the triangle.



To find p. Make p the middle part, and we have, by Napier's rules, or by (200),

$$\sin p = \sin A \ \sin h,$$

or, by logarithms,

$$\log \sin p = \log \sin A + \log \sin h. \qquad (208)$$

To find b. Make the complement of A the middle part, and we have, by Napier's rules, or by (202),

$$\cos A = \cot h \ \tan b;$$

whence, $$\tan b = \tan h \ \cos A, \qquad (209)$$

or, by logarithms,

$$\log \tan b = \log \tan h + \log \cos A. \qquad (210)$$

To find B. Make the complement of h the middle part, and we have, by Napier's rules, or by (199),

$$\cos h = \cot A \ \cot B;$$

whence, $$\cot B = \cos h \ \tan A, \qquad (211)$$

or, by logarithms,

$$\log \cot B = \log \cos h + \log \tan A. \qquad (212)$$

Thus, b and B, by observing the algebraic signs, are determined without ambiguity; and p, though determined by its sine, is not ambiguous, since it must be of the same species as A (Art. 170).

EXAMPLES.

1. Given in a right-angled spherical triangle $A\,B\,C$, right-angled at C, the hypothenuse h equal to 105° 34′, and the angle A equal to 80° 40′; to solve the triangle.

Solution.

By (208),	By (210),	By (212),
h, log sin $+9.983770$	log tan -10.555053	log cos $-\ 9.428717$
A, log sin $+9.994212$	log cos $+\ 9.209992$	log tan $+10.784220$
p, log sin $+9.977982$	b, log tan $-\ 9.765045$	B, log cot -10.212937

Hence, $p = 71° \ 54' \ 33''$, $b = 149° \ 47' \ 37''$, $B = 148° \ 30' \ 54''$.

2. Given in the spherical triangle $A\,B\,C$, right-angled at C,

the hypothenuse h equal to 70° 23′ 42″, and the angle A equal to 66° 20′ 40″; to find the other parts.

Ans. p, 59° 38′ 26″; b, 48° 24′ 15″; B, 52° 32′ 55″.

CASE II.

173. *Given the hypothenuse and one side.*

Let there be given (Fig. Art. 172) the hypothenuse h and the side p; to solve the triangle.

To find A. Make p the middle part, and we have, by Napier's rules, or by (200),

$$\sin p = \sin A \sin h;$$

whence,
$$\sin A = \frac{\sin p}{\sin h}, \tag{213}$$

or, by logarithms,

$$\log \sin A = \log \sin p - \log \sin h. \tag{214}$$

To find B. Make the complement of B the middle part, and we have, by Napier's rules, or by (203),

$$\cos B = \cot h \tan p,$$

or, by logarithms,

$$\log \cos B = \log \cot h + \log \tan p. \tag{215}$$

To find b. Make the complement of h the middle part, and we have, by Napier's rules, or by (198),

$$\cos h = \cos p \cos b;$$

whence,
$$\cos b = \frac{\cos h}{\cos p}, \tag{216}$$

or, by logarithms,

$$\log \cos b = \log \cos h - \log \cos p. \tag{217}$$

Here, as in the preceding article, b and B are determined without ambiguity, for there is only one angle less than 180° corresponding to a given cosine; and A must be of the same species as p.

EXAMPLES.

1. Given in a right-angled spherical triangle $A\,B\,C$, the hypothenuse h equal to 91° 42′, and the side p equal to 95° 22′ 30″; to solve the triangle.

Ans. A, 95° 6′; B, 71° 36′ 45″; b, 71° 32′ 12″.

2. Given in a right-angled spherical triangle, the hypothenuse equal to 70° 23′ and a side equal to 48° 24′; to solve the triangle.

CASE III.

174. *Given one side and its adjacent oblique angle.*

Let there be given (Fig. Art. 172) the side b and the angle A; to solve the triangle.

To find B. Make the complement of B the middle part, and we have, by Napier's rules, or by (207),

$$\cos B = \sin A \cos b,$$

or, by logarithms,

$$\log \cos B = \log \sin A + \log \cos b. \tag{218}$$

To find p. Make b the middle part, and we have, by Napier's rules, or by (205),

$$\sin b = \cot A \tan p;$$

whence,

$$\tan p = \tan A \sin b, \tag{219}$$

or, by logarithms,

$$\log \tan p = \log \tan A + \log \sin b. \tag{220}$$

To find h. Make the complement of A the middle part, and we have, by Napier's rules, or by (202),

$$\cos A = \cot h \tan b;$$

whence,

$$\cot h = \cos A \cot b, \tag{221}$$

or, by logarithms,

$$\log \cot h = \log \cos A + \log \cot b. \tag{222}$$

EXAMPLES.

1. Given in a spherical triangle $A\,B\,C$, right-angled at C, the side b equal to 29° 46′ 8″, and the angle A equal to 137° 24′ 21″; to solve the triangle.

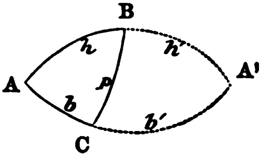
Ans. B, 54° 1′ 16″; p, 155° 27′ 54″; h, 142° 9′ 13″.

2. Given in a spherical triangle $A\,B\,C$, right-angled at C, the side p equal to 149° 47′ 23″, and the angle B equal to 80° 40′; to find the other parts.

CASE IV.

175. *Given one side and its opposite oblique angle.*

Let there be given in a spherical triangle $A\,B\,C$, right-angled at C, the side p and the opposite angle A; to solve the triangle.



To find h. Make p the middle part, and we have, by Napier's rules, or by (200),

$$\sin p = \sin A \sin h;$$

whence,
$$\sin h = \frac{\sin p}{\sin A}, \tag{223}$$

or, by logarithms,

$$\log \sin h = \log \sin p - \log \sin A. \tag{224}$$

To find b. Make b the middle part, and we have, by Napier's rules, or by (205),

$$\sin b = \cot A \tan p,$$

or, by logarithms,

$$\log \sin b = \log \cot A + \log \tan p. \tag{225}$$

To find B. Make the complement of A the middle part, and we have, by Napier's rules, or by (206),

$$\cos A = \sin B \cos p;$$

whence,
$$\sin B = \frac{\cos A}{\cos p}, \tag{226}$$

or, by logarithms,

$$\log \sin B = \log \cos A - \log \cos p. \tag{227}$$

Here, since all the unknown parts are determined by their sines, and since there are always two angles less than 180° corresponding to a given sine, h, b, and B may be taken either acute or obtuse; hence there may be two solutions.

For, produce $A\,B$ and $A\,C$ till they meet in A', then we have a second triangle, $A'\,B\,C$, which satisfies the given conditions, for it has a right angle at C, the given side p, and A' equal to A, the given angle. But h', b', and B, the other parts of the second triangle, are respectively the supplements of h, b, and B of the first triangle.

When, however, p is given equal to A, we have h, b, and B, each equal to 90°, and the triangle $A'B\,C$ is equal to the triangle $A\,B\,C$ (Geom., Prop. XII. Bk. IX).

When p and A are both equal to 90°, h is also equal to 90°, and b and B are equal, but indeterminate.

EXAMPLES.

1. Given in a spherical triangle $A\,B\,C$, right-angled at C, the side p equal to $36°\,31'$, and the angle A equal to $37°\,25'$; to solve the triangle.

Ans. h, $78°\ 20'$, or $101°\ 40'$; b, $75°\ 26'$, or $104°\ 34'$; B, $81°\ 12'$, or $98°\ 48'$, when carried only to minutes.

2. Given in a spherical triangle $A\,B\,C$, right-angled at C, the side b equal to $79°\,30'$, and the angle B equal to $89°\,35'$; to solve the triangle.

Case V.

176. *Given the two sides about the right angle.*

Let there be given (Fig. Art. 172) the sides p and b; to solve the triangle.

To find h. Make the complement of h the middle part, and we have, by Napier's rules, or by (198),

$$\cos h = \cos p \cos b,$$

or, by logarithms,

$$\log \cos h = \log \cos p + \log \cos b. \qquad (228)$$

To find A. Make b the middle part, and we have, by Napier's rules, or by (205),

$$\sin b = \cot A \ \tan p;$$

whence,
$$\cot A = \cot p \ \sin b, \qquad (229)$$
or, by logarithms,

$$\log \cot A = \log \cot p + \log \sin b. \qquad (230)$$

To find B. Make p the middle part, and we have, by Napier's rules, or by (204),

$$\sin p = \cot B \ \tan b;$$

whence
$$\cot B = \sin p \ \cot b, \qquad (231)$$
or, by logarithms,

$$\log \cot B = \log \sin p + \log \cot b. \qquad (232)$$

These formulæ determine h, A, and B without ambiguity.

EXAMPLES.

1. In a spherical triangle $A\,B\,C$ are given the sides about the right angle, p equal to 48° 24′ 15″, and b equal to 59° 38′ 27″; to solve the triangle.

Ans. h, 70° 23′ 42″; A, 52° 32′ 55″; B, 66° 20′ 40″.

2. Given in a right-angled spherical triangle, the side p equal to 95° 22′ 30″, and the side b equal to 71° 32′ 14″; to find the other parts.

CASE VI.

177. *Given the two oblique angles.*

Let there be given (Fig. Art. 172) the angles A and B; to solve the triangle.

To find h. Make the complement of h the middle part, and we have, by Napier's rules, or by (199),

$$\cos h = \cot A \ \cot B,$$

or, by logarithms,

$$\log \cos h = \log \cot A + \log \cot B. \qquad (233)$$

To find p. Make the complement of A the middle part, and we have, by Napier's rules, or by (206),

$$\cos A = \sin B \cos p\,;$$

whence,
$$\cos p = \frac{\cos A}{\sin B}, \tag{234}$$

or, by logarithms,
$$\log \cos p = \log \cos A - \log \sin B. \tag{235}$$

To find b. Make the complement of B the middle part, and we have, by Napier's rules, or by (207),

$$\cos B = \sin A \ \cos b\,;$$

whence,
$$\cos b = \frac{\cos B}{\sin A}, \tag{236}$$

or, by logarithms,
$$\log \cos b = \log \cos B - \log \sin A. \tag{237}$$

Here h, p, and b are determined without ambiguity.

EXAMPLES.

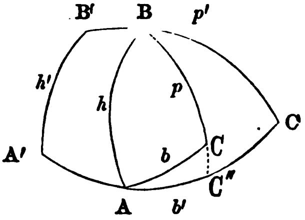
1. In a right-angled spherical triangle ABC are given the two oblique angles, A equal to 44° 50′, and B equal to 65° 49′ 53″; to solve the triangle.

Ans. h, 63° 10′ 4″; p, 38° 59′ 11″; b, 54° 30′.

2. Given, in a right-angled spherical triangle, the two oblique angles, A equal to 125° 30′, and B equal to 80° 40′; to find the other parts.

QUADRANTAL TRIANGLES.

178. A QUADRANTAL TRIANGLE is a spherical triangle having one of its sides quadrantal, or equal to 90°.



Quadrantal triangles may be solved in the same manner as right-angled spherical triangles, by means of the polar triangle.

Let ABC be a quadrantal triangle, and $A'B'C'$ denote a triangle polar to it; then, by Art. 150, we have

$$A' = 180° - p, \quad B' = 180° - b, \quad C' = 180° - h\,;$$
$$p' = 180° - A, \quad b' = 180° - B, \quad h' = 180° - C.$$

Now, if the side h be taken equal to 90°, its corresponding polar angle C' will also equal 90°; hence the polar triangle will be right-angled, and can be solved by application of the preceding formulæ for right-angled spherical triangles, and thus the required parts of the quadrantal triangle may be determined.

A triangle, one of whose sides is a quadrant, may also be solved by laying off a quadrant on one of the other sides, prolonged if necessary, and connecting this last point with the other extremity of the original quadrant by the arc of a great circle, thus making the original quadrantal triangle either the difference or the sum of a bi-quadrantal and a right-angled spherical triangle. Solving the latter solves the original triangle. Thus, AC'' measures B, $C A C'' = 90° - A$, $C C'' = 90° - p$, $A C C'' = 180° - C$, and solving the triangle $A C C''$ also solves the triangle $A B C$.

EXAMPLES.

1. Let there be given, in a quadrantal triangle ABC, the side h equal to 90°, the angle A equal to 54° 43′, and the angle B equal to 42° 12′; to find the other parts.

By taking the supplements of the given parts, we have in the polar triangle,

$$p' = 125° 17', \qquad b' = 137° 48',$$

whence A', B', and h' are determined as in Art. 176, and the supplements of these give the required parts of the quadrantal triangle. Ans. p, 64° 34′ 40″; b, 48° 0′ 16″; C, 115° 20′ 5″.

2. Given two sides of a quadrantal triangle equal to 72° 53′ and 51° 4′, to find the angle opposite to the quadrantal side.

Ans. 104° 24′ 21″.

SOLUTION OF OBLIQUE-ANGLED SPHERICAL TRIANGLES.

179. In the solution of oblique-angled spherical triangles, it is sometimes found convenient, especially in removing an ambiguity, to refer to one or more of the following propositions, of which the first four have been demonstrated in Book IX. of the Geometry.

I. *Any side of a spherical triangle is less than the sum of the other two.*

II. *The sum of the sides is less than* 360°.

III. *The sum of the angles is greater than* 180°.

IV. *The greater side is opposite the greater angle, and conversely.*

V. *Any angle is greater than the difference between* 180° *and the sum of the other two angles.*

VI. *A side which differs more from* 90° *than another side, is of the same species as its opposite angle.*

For, by (150), we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A;$$

whence,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c},$$

in which the denominator is always positive.

Then, if a differs more from 90° than b or than c, $\cos a$ is numerically greater than $\cos b$, or than $\cos c$, and we have

$$\cos a > \cos b \cos c;$$

hence, the sign of the numerator, and consequently the sign of $\cos A$, is the same as that of $\cos a$, that is, A and a are in the same quadrant.

VII. *An angle which differs more from* 90° *than another angle, is of the same species as its opposite side.*

For, by (153), we have

$$\cos A = \sin B \sin C \cos a - \cos B \cos C;$$

whence,

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C},$$

in which, if A differs more from 90° than B, or than C, $\cos A$ is numerically greater than $\cos B$, or than $\cos C$, and the sign of $\cos a$ is the same as that of $\cos A$, that is, a and A are in the same quadrant.

VIII. *When the sum of two sides is greater than, equal to, or less than* 180°, *the sum of the two opposite angles is the same.*

For, by means of (188),

$$\tan \tfrac{1}{2}(a+b) \cos \tfrac{1}{2}(A+B) = \tan \tfrac{1}{2} c \cos \tfrac{1}{2}(A-B),$$

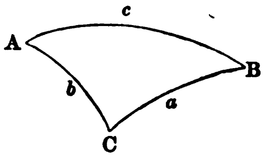
in which the second member is always positive, since $\frac{1}{2} c$ and $\frac{1}{2}(A - B)$ are each less than 90°, so that the factors of the first member, $\tan \frac{1}{2}(a+b)$ and $\cos \frac{1}{2}(A+B)$ must have the same sign. Therefore, $\frac{1}{2}(a+b)$ and $\frac{1}{2}(A+B)$ are of the same species.

180. In the solution of oblique-angled spherical triangles, there are six cases, the data in them being, respectively,

I. Two sides and an angle opposite one of them.
II. Two angles and a side opposite one of them.
III. Two sides and the included angle.
IV. Two angles and the included side.
V. The three sides.
VI. The three angles.

CASE I.

181. *Given two sides and an angle opposite one of them.*



Let there be given, in the oblique-angled spherical triangle ABC, the sides a and b, and the angle A; to solve the triangle.

To find B. We have, from (148),

$$\sin B = \frac{\sin b}{\sin a} \sin A,$$

or, by logarithms,

$$\log \sin B = \log \sin b - \log \sin a + \log \sin A. \qquad (238)$$

To find C and c. We have, by Napier's analogies, (186) and (188),

$$\cot \tfrac{1}{2} C = \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} \tan \tfrac{1}{2}(A+B),$$

$$\tan \tfrac{1}{2} c = \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} \tan \tfrac{1}{2}(a+b);$$

or, by logarithms,

$$\log\cot\tfrac{1}{2}C = \log\cos\tfrac{1}{2}(a+b) - \log\cos\tfrac{1}{2}(a-b) + \log\tan\tfrac{1}{2}(A+B), \qquad (239)$$

$$\log\tan\tfrac{1}{2}c = \log\cos\tfrac{1}{2}(A+B) - \log\cos\tfrac{1}{2}(A-B) + \log\tan\tfrac{1}{2}(a+b), \qquad (240)$$

which determine $\frac{1}{2}C$ and $\frac{1}{2}c$, and thence C and c.

In this case, since B is found from its sine, it will sometimes admit of two values, the one supplementary to the other. When B has two values, C and c must each have two corresponding values. Whether both values of B are admissible must be determined by one of the propositions of Art. 179.

Thus (Prop. VI.), if b differs more from 90° than a, B must be of the same species as b, and there can be but one solution; but if b differs less from 90° than a, there may be two solutions.

Or (Prop. VIII.), if only one of the supplementary values of B makes $\frac{1}{2}(A+B)$ of the same species as $\frac{1}{2}(a+b)$, there can be but one solution; but if both values of B fulfil that condition, there will be two solutions.

EXAMPLES.

1. Given, in an oblique-angled spherical triangle, the side a equal to 63° 50′, the side b equal to 80° 19′, and the angle A equal to 51° 30′; to solve the triangle.

Solution. By (238) we have

$a = 63° 50'$	ar. co. log sin	0.046958
$b = 80° 19'$	log sin	9.993768
$A = 51° 30'$	log sin	9.893544
$B = 59° 15' 57''$, or $120° 44' 3''$	log sin	9.934270

As b differs less from 90° than a, both values of B are admissible, and we have $\frac{1}{2}(b-a) = 8° 14' 30''$, $\frac{1}{2}(a+b) = 72° 4' 30''$, $\frac{1}{2}(A+B) = 55° 22' 58''$ or $86° 7' 2'$, and $\frac{1}{2}(B-A) = 3° 52' 58''$ or $34° 37' 2''$. The cosines of $\frac{1}{2}(b-a)$ and $\frac{1}{2}(B-A)$ are the same as those of $\frac{1}{2}(a-b)$ and $\frac{1}{2}(A-B)$, respectively, by Art. 79. Hence, by (239) and (240),

$\frac{1}{2}(b-a)$	ar. co. log cos +	0.004508		ar. co. log cos +	0.004508
$\frac{1}{2}(a+b)$	log cos +	9.488229		log cos +	9.488229
$\frac{1}{2}(A+B)$	log tan +	10.160964	or	log tan +	11.168314
$\frac{1}{2}C$	log cot +	9.653701	or	log cot +	10.661051

$$\tfrac{1}{2}C = 65^\circ\ 44'\ 53'' \text{ or } 12^\circ\ 18'\ 42'',$$

$$C = 131^\circ\ 29'\ 46'' \text{ or } 24^\circ\ 37'\ 24''.$$

$\frac{1}{2}(B-A)$	ar. co. log cos +	0.000998	or	ar. co. log cos +	0.084618
$\frac{1}{2}(A+B)$	log cos +	9.754418	or	log cos +	8.830687
$\frac{1}{2}(a+b)$	log tan +	10.490161		log tan +	10.490161
$\frac{1}{2}c$	log tan +	10.245577	or	log tan +	9.405466

$$\tfrac{1}{2}c = 60^\circ\ 23'\ 57'' \text{ or } 14^\circ\ 16'\ 18'',$$

$$c = 120^\circ\ 47'\ 54'' \text{ or } 28^\circ\ 32'\ 36''.$$

2. Given, in an oblique-angled spherical triangle, two sides equal to 99° 40′ 48″ and 64° 23′ 15″, and an angle opposite to the first of these equal to 95° 38′ 4″; to find the other side and angles.

Ans. Side, 100° 49′ 30″; angles, 65° 33′ 10″ and 97° 26′ 30″.

CASE II.

182. *Given two angles and a side opposite one of them.*

Let there be given, in the oblique-angled spherical triangle $A\,B\,C$ (Fig. Art. 181), the angles A and B, and the side a; to solve the triangle.

To find b. We have, from (148),

$$\sin b = \frac{\sin B}{\sin A} \sin a,$$

or, by logarithms,

$$\log \sin b = \log \sin B - \log \sin A + \log \sin a. \qquad (241)$$

To find C and c. We use equations (239) and (240), as in the last article.

This case is exactly analogous to Case I., and gives rise to the same ambiguities, as may be shown by passing to the polar triangle.

If B differs more from 90° than A, b must be of the same species as B, and there can be but one solution; but if B differs less from 90° than A, there may be two solutions. (Prop. VII. Art. 179.)

Or, if only one of the supplementary values of b makes $\frac{1}{2}(a+b)$ of the same species as $\frac{1}{2}(A+B)$, there can be but one solution; but if both values of b fulfil that condition, there will be two solutions. (Prop. VIII. Art. 179.)

EXAMPLES.

1. Given, in an oblique-angled spherical triangle, the angle A equal to 135°, the angle B equal to 60°, and the side a equal to 155°; to find the other parts.

Ans. C, 98° 3′ 4″ or 16° 57′ 1″; b, 31° 10′ 17″ or 148° 49′ 43″; c, 143° 42′ 57″ or 10° 2′ 6″.

2. Given, in an oblique-angled spherical triangle, two angles equal to 97° 26′ 30″ and 65° 33′ 10″, and the side opposite to the first equal to 100° 49′ 30″; to find the other parts.

Case III.

183. *Given two sides and the included angle.*

Let there be given, in the oblique-angled spherical triangle ABC (Fig. Art. 181), the sides a and b, and the included angle C; to solve the triangle.

To find A and B. By means of Napier's analogies (186) and (187), we have

$$\tan\tfrac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C,$$

$$\tan\tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C;$$

or, by logarithms,

$$\log\tan\tfrac{1}{2}(A+B) = \log\cos\tfrac{1}{2}(a-b) - \log\cos\tfrac{1}{2}(a+b) + \log\cot\tfrac{1}{2}C, \tag{242}$$

$$\log\tan\tfrac{1}{2}(A-B) = \log\sin\tfrac{1}{2}(a-b) - \log\sin\tfrac{1}{2}(a+b) + \log\cot\tfrac{1}{2}C, \tag{243}$$

which determine $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$. The sum of these values gives A, and the second subtracted from the first gives B.

To find c. We use equation (240), as in the first two cases. The value of c might also be obtained by (147) or (149); but as it is thus determined from its sine, it would be necessary to remove the ambiguity by means of the principles contained in Art. 179.

As A, B, and c may all be found by means of tangents, there can be but one value for each. It will be observed that $\frac{1}{2}(A+B)$ must always be of the same species as $\frac{1}{2}(a+b)$. (Prop. VIII. Art. 179.)

EXAMPLES.

1. Given, in an oblique-angled spherical triangle $A\,B\,C$, the side a equal to 70°, the side b equal to 38° 30′, and the included angle C equal to 31° 34′ 26″; to solve the triangle.

Solution.

$\frac{1}{2}(a+b) = 54° \; 15'$, $\frac{1}{2}(a-b) = 15° \; 45'$, and $\frac{1}{2}C = 15° \; 47' \; 13''$; then,

	By (242),			By (243),	
$\frac{1}{2}(a+b)$	ar. co. log cos $+$	0.233402		ar. co. log sin $+$	0.090672
$\frac{1}{2}(a-b)$	log cos $+$	9.983381		log sin $+$	9.433675
$\frac{1}{2}C$	log cot $+$	10.548635		log cot $+$	10.548635
$\frac{1}{2}(A+B)$	log tan $+$	10.765418	$\frac{1}{2}(A-B)$	log tan $+$	10.072982

$\frac{1}{2}(A+B) = 80° \; 15' \; 41''$ $\qquad$ $\frac{1}{2}(A-B) = 49° \; 47' \; 30''$

$A = 130° \; 3' \; 11''$ $\qquad$ $B = 30° \; 28' \; 11''$

By (240),

$\frac{1}{2}(A-B)$	$= 49° \; 47' \; 30''$	ar. co. log cos $+$	0.190058
$\frac{1}{2}(A+B)$	$= 80° \; 15' \; 41''$	log cos $+$	9.228282
$\frac{1}{2}(a+b)$	$= 54° \; 15'$	log tan $+$	10.142730
$\frac{1}{2}c$	$= 20°$	log tan $+$	9.561070

Ans. Angle A, 130° 3′ 11″; angle B, 30° 28′ 11″; side c, 40°.

2. Given, in an oblique-angled spherical triangle, an angle equal to 48° 36′, and the two adjacent sides equal to 112° 22′ 58½″ and 89° 16′ 53½″; to find the other parts.

CASE IV.

184. *Given two angles and the included side.*

Let there be given, in the oblique-angled spherical triangle ABC (Fig. Art. 181), the angles A and B, and the included side c; to solve the triangle.

To find a and b. By means of Napier's analogies (188) and (189), we have

$$\tan \tfrac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \tfrac{1}{2} c,$$

$$\tan \tfrac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \tfrac{1}{2} c;$$

or, by logarithms,

$$\log \tan \tfrac{1}{2}(a+b) = \log \cos \tfrac{1}{2}(A-B) - \log \cos \tfrac{1}{2}(A+B) + \log \tan \tfrac{1}{2} c, \qquad (244)$$

$$\log \tan \tfrac{1}{2}(a-b) = \log \sin \tfrac{1}{2}(A-B) - \log \sin \tfrac{1}{2}(A+B) + \log \tan \tfrac{1}{2} c, \qquad (245)$$

which determine $\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$, and thence a and b.

To find C. We use equation (239), as in the first two cases; but (147) or (149) may be employed, as in the last case.

This case is analogous to Case III., and gives rise to no ambiguity.

EXAMPLES.

1. Given, in an oblique-angled spherical triangle ABC, the angles A and B equal to 119° 15′ and 70° 39′, and the side c equal to 52° 39′ 4″; to solve the triangle.

Ans. Sides a and b, 112° 22′ 58½″ and 89° 16′ 53½″; angle C, 48° 36′.

2. In an oblique-angled spherical triangle, given two angles equal to 130° 3′ 11″ and 31° 34′ 26″, and the included side equal to 38° 30′; to find the other parts.

CASE V.

185. *Given the three sides.*

Let there be given, in the oblique-angled spherical triangle ABC (Fig. Art. 181), the sides a, b, and c; to solve the triangle.

To find A, B, and C, we have, by (164), (165), and (166),

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}},$$

$$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s-c) \sin (s-a)}{\sin c \sin a}},$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin a \sin b}};$$

or, by logarithms,

$$\log \sin \tfrac{1}{2} A = \frac{\log \sin (s-b) + \log \sin (s-c) - \log \sin b - \log \sin c}{2}, \quad (246)$$

$$\log \sin \tfrac{1}{2} B = \frac{\log \sin (s-c) + \log \sin (s-a) - \log \sin c - \log \sin a}{2}, \quad (247)$$

$$\log \sin \tfrac{1}{2} C = \frac{\log \sin (s-a) + \log \sin (s-b) - \log \sin a - \log \sin b}{2}. \quad (248)$$

A, B, and C can also be determined by formulæ (167), (168), and (169) for the cosine of half an angle, and by formulæ (170), (171), and (172) for the tangent of half an angle.

Since the half-angles must be less than 90°, there is no ambiguity in determining the angles by any of these formulæ.

EXAMPLES.

1. Given, in an oblique-angled spherical triangle, the side a equal to 70°, the side b equal to 38°, and the side c equal to 40°; to find the angles.

Solution.

$$s = 74^\circ, \quad s-a = 4^\circ, \quad s-b = 36^\circ, \quad s-c = 34^\circ.$$

	By (246),	By (247),	By (248),
$s-b$,	log sin 9.769219		log sin 9.769219
$s-c$,	log sin 9.747562	log sin 9.747562	
$s-a$,		log sin 8.843585	log sin 8.843585
b,	ar.co.log sin 0.210658		ar.co.log sin 0.210658
c,	ar.co.log sin 0.191933	ar.co.log sin 0.191933	
a,		ar.co.log sin 0.027014	ar.co.log sin 0.027014
	2) 19.919372	2) 18.810094	2) 18.850476
	$\frac{1}{2} A$, log sin 9.959686	$\frac{1}{2} B$, log sin 9.405047	$\frac{1}{2} C$, log sin 9.425238

$\frac{1}{2}A = 65^\circ\,41'\,33''.7 \quad \frac{1}{2}B = 14^\circ\,43'\,18'' \quad \frac{1}{2}C = 15^\circ\,26'\,21''.7$

Ans. A, 131° 23′ 7″; B, 29° 26′ 36″; C, 30° 52′ 43″.

2. Given, in an oblique-angled spherical triangle, the sides equal to 112° 22′ 59″, 89° 16′ 53″, and 52° 39′ 4″; to solve the triangle.

Case VI.

186. *Given the three angles.*

Let there be given, in the oblique-angled spherical triangle ABC (Fig. Art. 181), the angles A, B, and C; to solve the triangle.

To find a, b, and c, we have, by (175),

$$\sin \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}},$$

$$\sin \tfrac{1}{2} b = \sqrt{\frac{-\cos S \cos (S-B)}{\sin C \sin A}},$$

$$\sin \tfrac{1}{2} c = \sqrt{\frac{-\cos S \cos (S-C)}{\sin A \sin B}};$$

or, by logarithms,

$$\log \sin \tfrac{1}{2} a = \frac{\log \cos S + \log \cos (S-A) - \log \sin B - \log \sin C}{2}, \quad (249)$$

$$\log \sin \tfrac{1}{2} b = \frac{\log \cos S + \log \cos (S-B) - \log \sin C - \log \sin A}{2}, \quad (250)$$

$$\log \sin \tfrac{1}{2} c = \frac{\log \cos S + \log \cos (S-C) - \log \sin A - \log \sin B}{2}. \quad (251)$$

a, b, and c can also be determined by formulæ (176) for the cosine of half an angle, and by formulæ (177) for the tangent of half an angle.

Here a, b, and c are determined without ambiguity.

EXAMPLES.

1. Given, in an oblique-angled spherical triangle, the angle A equal to 120° 43′ 37″, the angle B equal to 109° 55′ 42″, and the angle C equal to 116° 38′ 33″; to find the sides.

Ans. a, 115° 13′ 26″; b, 98° 21′ 39″; c, 109° 50′ 20″.

2. Given the angles in an oblique-angled spherical triangle equal to 131° 23′ 7″, 29° 26′ 36″, and 30° 52′ 43″; to solve the triangle.

BOOK VI.

APPLICATIONS OF SPHERICAL TRIGONOMETRY TO ASTRONOMY AND GEOGRAPHY.

187. The **Celestial Sphere** is the spherical concave surrounding the earth, in which all the heavenly bodies appear to be situated.

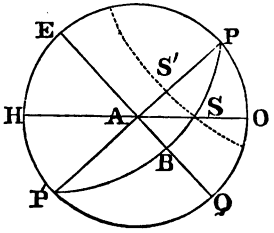
188. The **Zenith** is that pole of the horizon which is directly overhead.

189. The **Altitude** of a heavenly body is its distance above the horizon, measured on the arc of a great circle passing through that body and the zenith.

190. The **Declination** of a heavenly body is its distance north or south of the celestial equator, measured on a meridian.

191. *The altitude of the celestial pole is equal to the latitude of the place where the observer is located.*

For the distance from the zenith to the celestial equator is the latitude of the place, and the distance from the zenith to the pole is its complement; but the distance from the zenith to the pole is also the complement of the altitude of the pole; hence the latitude of the place and the altitude of the pole are equal.



192. *To find the time of the* RISING AND SETTING OF THE SUN *at any place, the sun's declination and the latitude of the place being given.*

Let P represent the celestial north pole, $E\,A\,Q$ the celestial equator, $H\,A\,O$ the rational horizon, S the place of the sun's rising, S' the position of the sun at 6 o'clock, $P E P'$ the meridian of the given place, $P B P'$ the meridian passing through S, and $P A P'$ the meridian 90° distant from $P E P'$, passing through S'.

From the time of the sun's rising to 6 o'clock, it will pass over $S S$, the arc of a small circle, corresponding to $B A$, the arc of a great circle. The length of $B A$, expressed in time (Art. 147), will then give the amount to be taken from or added to 6 o'clock, to give the time of the sun's rising or setting.

$B S$ is the sun's declination, $P O$ is the latitude of the place (Art. 191), and $Q O$, which measures the angle $B A S$, is its complement; hence, in the right-angled spherical triangle $A B S$, there are known the side $B S$ and the angle $B A S$, from which, by Art. 175,

$$\sin B A = \tan B S \cot B A S,$$

or, $\log \sin B A = \log \tan$ sun's decl. $+ \log \tan$ lat. of place.

After reducing the arc $B A$ to time, at the rate of 15° to an hour, or 4m. to a degree, it must be added to 6 o'clock for the time of the sun's setting, and subtracted for its rising, when the declination and latitude are both north or both south; but subtracted for its setting and added for its rising, when one is north and the other south.

The preceding reasoning rests upon the assumption that the sun's declination does not change between sunrise and sunset, which, although not strictly true, is accurate enough for our present purpose. The time obtained is *apparent time*, and a correction must be applied if we wish to find mean time, or that indicated by the clock. Another correction is necessary for refraction. Neither of these corrections has, however, been applied to the answers that follow.

EXAMPLES.

1. Required the time of the sun's rising and setting in Edinburgh, latitude 55° 57′ N., when the sun's declination is 23° 28′ N.

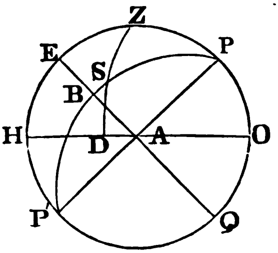
Ans. Rises, 3h. 20m. 7s.; sets, 8h. 39m. 53s.

2. What is the time of the sun's rising and setting in latitude 60° 3′ N., when the sun's declination is 23° 28′ S.?

Ans. Rises, 9h. 15m. 33s.; sets, 2h. 44m. 27s.

3. Required the time of the sun's rising and setting in places whose latitude is 48° S., when the sun's declination is 15° S.

193. *To find the* HOUR OF THE DAY *at any place, the latitude of the place and the sun's declination and altitude being given.*



Let Z represent the zenith, and $Z\,D$ an arc of a great circle drawn through the zenith and the sun's place, S; $P\,B\,P'$, a meridian drawn through the sun's place, &c., as in the last article.

As before, the arc $A\,B$, added to or subtracted from 6 o'clock, will give the time when the sun is at S; but it will be more convenient to use its complement, $E\,B$, which is the time before or after 12 o'clock.

$E\,Z$ is the latitude of the place, and $P\,Z$ is its complement; $B\,S$ is the sun's declination, and $P\,S$ is its complement; $S\,D$ is the altitude of the sun, and $Z\,S$ is its complement; hence, in the spherical triangle $Z\,P\,S$, the three sides are known, and the angle $Z\,P\,S$, or the arc $E\,B$, may be found by Art. 185.

If either the sun's declination, or the latitude of the place, is south, it must be considered negative in taking its complement, unless the south pole is taken as a vertex of the triangle, when north will be negative.

EXAMPLES.

1. Required the apparent time of day in the morning, at a place in latitude 39° 54′ N., the sun's declination being 17° 29′ N., and its corrected altitude 15° 54′.

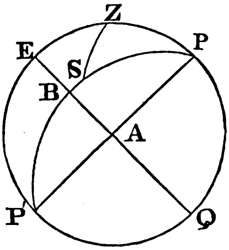
Ans. 6h. 25m. 30s. A. M.

2. In latitude 36° 39′ S., when the sun's declination was 9° 27′ N., its corrected altitude was observed, in the afternoon, to be 10° 40′; what was the apparent solar time?

Ans. 4h. 36m. 10s. P. M.

3. Required the apparent time of day in Boston, latitude 42° 21′ N., when the sun's declination is 20° S., and its corrected altitude 15° 15′, the sun being east of the meridian.

194. *To find the* SHORTEST DISTANCE *between two places on the earth's surface, and the* BEARING *of one from the other, their latitudes and longitudes being given.*



Let *Z* and *S* represent the two points on the earth's surface, and *P* the north pole of the earth. *P Z* and *P S* are the complements of the latitudes of the two places, and the arc *EB*, or the angle *ZP S*, is the difference of their longitudes; hence, in the spherical triangle *S P Z*, the two sides *P Z* and *P S*, and their included angle *P*, are known, from which the side *Z S*, and the angles *Z S P* and *S Z P* may be found by Art. 183.

The distance *Z S* can easily be reduced to miles by allowing 69.16 statute miles, or 60 nautical miles, to a degree. The answers which follow are given in statute miles.

If one place is south and the other north of the equator, the south latitude must be considered negative in taking its complement.

EXAMPLES.

1. What is the distance and bearing of Jerusalem, lat. 31° 47′ N., long. 35° 20′ E., from London, lat. 51° 30′ N., long. 6′ W.?

Ans. Distance, 2248 miles; bearing, S. 66° 31′ E.

2. Required the distance and bearing of Cape Horn, lat. 55° 58′ S., long. 67° 21′ W., from London.

Ans. Distance, 8363 miles; bearing, S. 36° 59′ W.

3. Required the distance and bearing of Quito, lat. 0°, long. 78° 45′ W., from San Francisco, lat. 37° 49′ N., long. 122° 14′ W.

A

TABLE,

CONTAINING THE

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

Numbers from 1 to 100 and their Logarithms, with their Indices.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

NOTE.—In the following part of the Table the Indices are omitted, as they can be very easily supplied by the directions given in the Author's NEW HIGHER ALGEBRA, p. 315, or Book I. of his TRIGONOMETRY

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8706	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
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9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
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3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
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6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
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5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
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7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
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4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
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7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
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6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
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8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
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2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
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4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
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6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
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3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
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5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
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7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
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7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
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6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
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6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
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4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
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6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
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7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
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9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
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4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
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9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
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5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
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2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
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4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
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5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
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9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
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6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
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2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
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2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
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5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
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1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
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6	7898	7972	8046	8120	~~8194~~	~~8268~~	~~8342~~	~~8416~~	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9672	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
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2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
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2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
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2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
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7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
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5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
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7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
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1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
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7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
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9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
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5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
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2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
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9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
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5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
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9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
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5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
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3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
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9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
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3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
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6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
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1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
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3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
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7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
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2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
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3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897959	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	969569	939619	939669	939719	939763	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3445	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	960042	960090	960138	960185	960233	960281	960328	960376	960423	48
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
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940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
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A

TABLE

OF LOGARITHMIC

SINES, COSINES, TANGENTS, AND COTANGENTS,

FOR EVERY

DEGREE AND MINUTE OF THE

QUADRANT.

M.	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	
0	0.000000		10.000000		0.000000		Infinite.	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	13.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.83	.235244	58
3	.940847	2082.31	.000000	.00	.940847	2082.31	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	12.934214	56
5	.162696	1319.68	.000000	.00	.162696	1319.69	.837304	55
6	.241877	1115.75	9.999999	.01	.241878	1115.78	.758122	54
7	.308824	966.53	.999999	.01	.308825	996.53	.691175	53
8	.366816	852.54	.999999	.01	.366817	852.54	.633183	52
9	.417968	762.63	.999999	.01	.417970	762.63	.582030	51
10	.463725	689.88	.999998	.01	.463727	689.88	.536273	50
11	7.505118	629.81	9.999998	.01	7.505120	629.81	12.494880	49
12	.542906	579 36	.999997	.01	.542909	579.33	.457091	48
13	.577668	536.41	.999997	.01	.577672	536.42	.422328	47
14	.609853	499.38	.999996	.01	.609857	499.39	.390143	46
15	.639816	467.14	.999996	.01	.639820	467.15	.360180	45
16	.667845	438.81	.999995	.01	.667849	438.82	.332151	44
17	.694173	413.72	.999995	.01	.694179	413.73	.305821	43
18	.718997	391.35	.999994	.01	.719004	391.36	.280997	42
19	.742477	371.27	.999993	.01	.742484	371.28	.257516	41
20	.764754	353.15	.999993	.01	.764761	351.36	.235239	40
21	7.785943	336.72	9.999992	.01	7 785951	336.73	12.214049	39
22	.806146	321.75	.999991	.01	.806155	321.76	.193845	38
23	.825451	308.05	.999990	.01	.825460	308.06	.174540	37
24	.843934	295.47	.999989	.02	.843944	295.49	.156056	36
25	.861662	283.88	.999988	.02	.861674	283.90	.138326	35
26	.878695	273.17	.999988	.02	.878708	273.18	.121292	34
27	.895085	263.23	.999987	.02	.895099	263.25	.104901	33
28	.910879	253.99	.999986	.02	.910894	254.01	.089106	32
29	.926119	245.38	.999985	.02	.926134	245.40	.073866	31
30	.940842	237.33	.999983	.02	.940858	237.35	.059142	30
31	7.955082	229.80	9.999982	.02	7.955100	229.81	12.044900	29
32	.968870	222.73	.999981	.02	.968889	222.75	.031111	28
33	.982233	216.08	.999980	.02	.982253	216.10	.017747	27
34	.995198	209.81	.999979	.02	.995219	209.83	.004781	26
35	8.007787	203.90	.999977	.02	8.007809	203 92	11.992191	25
36	.020021	198.31	.999976	.02	.020045	198.33	.979955	24
37	.031919	193.02	.999975	.02	.031945	193.05	.968055	23
38	.043501	188.01	.999973	.02	.043527	188.03	956473	22
39	.054781	183.25	.999972	.02	.054809	183.27	.945191	21
40	.065776	178.72	.999971	.02	.065806	178.74	.934194	20
41	8.076500	174.41	9.999969	.02	8.076531	174.44	11.923469	19
42	.086965	170.31	.999968	.02	.086997	170 34	.913003	18
43	.097183	166.39	.999966	.02	.097217	166.42	.902783	17
44	.107167	162.65	.999964	.03	.107202	162.68	.892797	16
45	.116926	159.08	.999963	.03	.116963	159.10	.883037	15
46	.126471	155.66	.999961	.03	.126510	155.68	.873490	14
47	.135810	152.38	.999959	.03	.135851	152.41	.864149	13
48	.144953	149.24	.999958	.03	.144996	149.27	.855004	12
49	.153907	146.22	.999956	.03	.153952	146.27	.846048	11
50	.162681	143.33	.999954	.03	.162727	143.36	.837273	10
51	8.171280	140.54	9.999952	.03	8 171328	140.57	11.828672	9
52	.179713	137.86	.999950	.03	.179763	137.90	.820237	8
53	.187985	135.29	.999948	.03	.188036	135.32	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.84	.803844	6
55	.204070	130.41	.999944	.03	.204126	130.44	.795874	5
56	.211895	128.10	.999942	.04	.211953	128.14	.788047	4
57	.219581	125.87	.999940	.04	.219641	125.90	.780359	3
58	.227134	123.72	.999938	.04	.227195	123.76	.772805	2
59	.234557	121.64	.999936	.04	.234621	121.68	.765379	1
60	.241855	119.63	.999934	.04	.241921	119.67	.758079	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	.249033	117.68	.999932	.04	.249102	117.72	.750898	59
2	.256094	115.80	.999929	.04	.256165	115.84	.743835	58
3	.263042	113.98	.999927	.04	.263115	114.02	.736885	57
4	.269881	112.21	.999925	.04	.269956	112.25	.730044	56
5	.276614	110.50	.999922	.04	.276691	110.54	.723309	55
6	.283243	108 83	.999920	.04	.283323	108.87	.716677	54
7	.289773	107.21	.999918	.04	.289856	107.26	.710144	53
8	.296207	105.65	.999915	.04	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.04	.302634	104.18	.697366	51
10	.308794	102.66	.999910	.04	.308884	102.70	.691116	50
11	8.314904	101.22	9.999907	.04	8.315046	101.26	11.684954	49
12	.321027	99.82	.999905	.04	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.04	.327114	98.51	.672886	47
14	.332924	97.14	.999899	.05	.333025	97.19	.666975	46
15	.338753	95.86	.999897	.05	.338856	95.90	.661144	45
16	.344504	94 60	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.38	.999891	.05	.350289	93.43	.649711	43
18	.355783	92 19	.999888	.05	.355895	92.24	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	.366777	89.90	.999882	.05	.366895	89.95	.633105	40
21	8.372171	88.80	9.999879	.05	8.372292	88.85	11.627708	39
22	.377499	87 72	.999876	.05	.377622	87.77	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.64	.999870	.05	.388092	85.70	.611908	36
25	.393101	84.64	.999867	.05	.393234	84.70	.606766	35
26	.398179	83.66	.999864	.05	.398315	83.71	.601685	34
27	.403199	82 71	.999861	.05	.403338	82 76	.596662	33
28	.408161	81.77	.999858	.05	.408304	81.82	.591696	32
29	.413068	80.86	.999854	.05	.413213	80.91	.586787	31
30	.417919	79.96	.999851	.06	.418068	80.02	.581932	30
31	8.422717	79.09	9.999848	.06	8.422869	79.14	11.577131	29
32	.427462	78.23	.999844	.06	.427618	78.30	.572382	28
33	.432156	77.40	.999841	.06	432315	77.45	.567685	27
34	.436800	76.57	.999838	.06	.436962	76.63	.563038	26
35	.441394	75.77	.999834	.06	.441560	75.83	.558440	25
36	.445941	74.99	.999831	.06	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.06	.450613	74.28	.549387	23
38	.454893	73.46	.999823	.06	.455070	73 52	.544930	22
39	.459301	72.73	.999820	.06	.459481	72.79	.540519	21
40	.463665	72.00	.999816	.06	.463849	72.06	.536151	20
41	8.467985	71.29	9.999812	.09	8.468172	71.35	11.531828	19
42	.472263	70.60	.999809	.06	.472454	70.66	.527546	18
43	.476498	69.91	.999805	.06	.476693	69.98	.523307	17
44	.480693	69.24	.999801	.06	.480892	69.31	.519108	16
45	.484848	68.59	.999797	.07	.485050	68 65	.514950	15
46	.488963	67.94	.999793	.07	.489170	68.01	.510830	14
47	.493040	67.31	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.69	.999786	.07	.497293	66.76	.502707	12
49	.501080	66.08	.999782	.07	.501298	66.15	.498702	11
50	505045	65.48	.999778	07	.505267	65.55	.494733	10
51	8.508974	64.89	9.999774	.07	8.509200	64.96	11.490800	9
52	.512867	64 31	.999769	.07	.513098	64.39	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.19	.999761	.07	.520790	63.26	.479210	6
55	.524343	62 64	.999757	.07	.524586	62.72	.475414	5
56	.528102	62.11	.999753	.07	.528349	62.18	.471651	4
57	.531828	61.58	.999748	.07	.532080	61.65	.467920	3
58	.535523	61.06	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.07	.539447	60.62	.460553	1
60	.542819	60.04	.999735	.07	.543084	60.12	.456916	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.07	.546691	59.62	.453309	59
2	.549995	59.06	.999726	.07	.550268	59.14	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.66	.446183	57
4	.557054	58.11	.999717	.08	.557336	58.19	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.73	.439172	55
6	.563999	57.19	.999708	.08	.564291	57.27	.435709	54
7	.567431	56.74	.999704	.08	.567727	56.82	.432273	53
8	.570836	56 30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.44	.999689	.08	.577877	55.52	.422123	50
11	8.580892	55 02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.19	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.79	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.39	.999665	.08	.594283	53.47	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.61	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.86	.999645	.09	.606978	51.94	.393022	41
20	.609734	51.49	.999640	.09	.610094	51.58	.389906	40
21	8.612823	51.12	9.999635	.09	8.613189	51.21	11 386811	39
22	.615891	50.76	.999629	.09	.616262	50.85	.383738	38
23	.618937	50.41	.999624	.09	.619313	50.50	.380687	37
24	.621962	50.06	.999619	.09	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.09	.625352	49.81	.374648	35
26	.627948	49.38	.999608	.09	.628340	49 47	.371660	34
27	.630911	49.04	.999603	.09	.631308	49.13	.368692	33
28	.633854	48.71	.999597	.09	.634256	48.80	.365744	32
29	.636776	48.39	.999592	.09	.637184	48.48	.362816	31
30	.639680	48.06	.999586	.09	.640093	48.16	.359907	30
31	8.642563	47.75	9.999581	.09	8.642982	47.84	11.357018	29
32	.645428	47.43	.999575	09	.645853	47.53	.354147	28
33	.648274	47.12	.999570	.09	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.09	.651537	46.91	.348463	26
35	.653911	46.52	.999558	.10	.654352	46.61	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.31	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.44	.334567	21
40	.667689	45.06	.999529	.10	.668160	45.26	.331840	20
41	8.670393	44.79	9.999524	.10	8.670870	44 88	11 329130	19
42	.673080	44.51	.999518	.10	.673563	44.61	.326437	18
43	.675751	44.24	.999512	.10	.676239	44.34	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.17	.321100	16
45	.681043	43.70	.999500	.10	.681544	43.80	.318456	15
46	.683665	43.44	.999493	.10	.684172	43.54	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.52	.305471	10
51	8 696543	42.17	9.999463	.11	8 697081	42.28	11.302919	9
52	.699073	41.92	.999456	.11	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.11	.702139	41.79	.297861	7
54	.704090	41.44	.999443	.11	.704646	41.55	.295354	6
55	.706577	41.21	.999437	.11	.707140	41.32	.292860	5
56	.709049	40.97	.999431	.11	.709618	41.08	.290382	4
57	.711507	40.74	.999424	.11	.712083	40.85	.287917	3
58	.713952	40.51	.999418	.11	.714534	40.62	.285465	2
59	.716383	40.29	.999411	.11	.716972	40.40	.283028	1
60	.718800	40.06	.999404	.11	.719396	40.17	.280604	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

87°

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	8.718800	40.06	9.999404	.11	8.719396	40.17	11.280604	60
1	.721204	39.84	.999398	.11	.721806	39.95	.278194	59
2	.723595	39.62	.999391	.11	.724204	39.74	.275796	58
3	.725972	39.41	.999384	.11	.726588	39.52	.273412	57
4	.728337	39.19	.999378	.11	.728959	39.30	.271041	56
5	.730688	38 98	.999371	.11	.731317	39.09	.268683	55
6	.733027	38.77	.999364	.12	.733663	38.89	.266337	54
7	.735354	38.57	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.36	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.16	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.96	.999336	.12	.742922	38.07	.257078	50
11	8.744536	37.76	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.56	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.49	.250260	47
14	.751297	37.17	.999308	.12	.751989	37.29	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.79	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.61	.999286	.12	.758668	36.73	.241332	43
18	.760151	36.42	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.24	.999272	.12	.763065	36.36	.236935	41
20	.764511	36.06	.999265	.12	.765246	36.18	.234754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.00	11.232583	39
22	.768828	35 70	.999250	.13	.769578	35.83	.230422	38
23	.770970	35.53	.999242	.13	.771727	35.65	.228273	37
24	.773101	35.35	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.18	.999227	.13	.775995	35.31	.224005	35
26	.777333	35.01	.999220	.13	.778114	35.14	.221886	34
27	.779434	34.84	.999212	.13	.780222	34.97	.219778	33
28	.781524	34.67	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.51	.999197	.13	.784408	34 64	.215592	31
30	.785675	34.31	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9.999181	.13	8.788554	34.31	11.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.86	.999166	.13	792662	33.99	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.54	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.39	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.23	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.08	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.78	11.191283	19
42	.809777	32.49	.999094	.14	.810683	32.62	.189317	18
43	.811726	32.34	.999086	.14	.812641	32.48	.187359	17
44	.813667	32.19	.999077	.14	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.14	.816529	32.19	.183471	15
46	.817522	31.91	.999061	.14	.818461	32.05	.181539	14
47	.819436	31.77	.999053	.14	.820384	31.91	.179616	13
48	.821343	31.63	.999044	.14	.822298	31.77	.177702	12
49	.823240	31.49	.999036	.14	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.14	.826103	31.50	.173897	10
51	8.827011	31 22	9.999019	.14	8.827992	31.36	11.172008	9
52	.828884	31.08	.999010	.14	.829874	31.23	.170126	8
53	.830749	30.95	.999002	.14	.831748	31.10	.168252	7
54	.832607	30.82	.998993	.14	.833613	30.96	.166387	6
55	.834456	30.69	.998984	.14	.835471	30.83	.164529	5
56	.836297	30.56	.998976	.14	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.57	.160837	3
58	.839956	30.30	.998958	.15	.840998	30.45	.159002	2
59	.841774	30.17	.998950	.15	.842825	30.32	.157175	1
60	.843585	30.00	.998941	.15	.844644	30.19	.155356	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M	Sine.	D	Cosine.	D.	Tang.	D..	Cotang.	
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	.845387	29.92	.998932	.15	.846455	30.07	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.55	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.31	.998887	.15	.855403	29.46	.144597	54
7	.856049	29.19	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.07	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.96	.998860	.15	.860686	29.11	.139314	51
10	.861283	28.84	.998851	.15	.862433	29.00	.137567	50
11	8.863014	28.73	9.998841	.15	8.864178	28.88	11.135827	49
12	.864738	28.61	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.16	.867632	28.66	.132368	47
14	.868165	28.39	.998813	.16	.869351	28.54	.130649	46
15	.869868	28.28	.998804	.16	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.16	.872770	28.32	.127230	44
17	.873255	28.06	.998785	.16	.874469	28.21	.125531	43
18	.874938	27.95	.998776	.16	.876162	28.11	.123838	42
19	.376615	27.86	.998766	.16	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.16	.879529	27.89	.120471	40
21	8.879949	27.63	9.998747	.16	8.881202	27.79	11.118798	39
22	.881607	27.52	.998738	.16	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.16	.884530	27.58	.115470	37
24	.884903	27.31	.998718	.16	.886185	27.47	.113815	36
25	.886542	27.21	.998708	.16	.887833	27.37	.112167	35
26	.888174	27.11	.998699	.16	.889476	27 27	.110524	34
27	.889801	27.00	.998689	.16	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.16	.892742	27.07	.107258	82
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.70	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.77	11.102404	29
32	.897842	26.51	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.41	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.31	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.29	.094430	24
37	.905736	26.03	.998589	.17	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.84	.998568	.17	.910285	26.01	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.66	9.998548	.17	8.913401	25.83	11.086599	19
42	.913488	25.56	.998537	.17	.914951	25.74	.085049	18
43	.915022	25.47	.998527	.17	.916495	25.65	.083505	17
44	.916550	25.38	.998516	.18	.918034	25.56	.081966	16
45	.918073	25.29	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.18	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.30	.077381	13
48	.922610	25.03	.998474	.18	.924136	25.21	.075864	12
49	.924112	24.94	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.86	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.77	9.998442	.18	8.928658	24 95	11.071342	9
52	.928587	24.69	.998431	.18	.930155	24.86	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.61	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	8
58	.937398	24.19	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.11	.998355	.18	.940494	24.30	.059506	1
60	.940296	24.03	.998344	.18	.941952	24.21	.058048	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	8.940296	24.03	9.998344	.19	8.941952	24.21	11.058048	60
1	.941738	23.94	.998333	.19	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.19	.944852	24.05	.055148	58
3	.944606	23.79	.998311	.19	.946295	23.97	.053705	57
4	.946034	23.71	.998300	.19	.947734	23.90	.052266	56
5	.947456	23 63	.998289	.19	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.19	.950597	23.74	.049403	54
7	.950287	23.48	.998266	.19	.952021	23.66	.047979	53
8	.951696	23.40	.998255	.19	.953441	23.60	.046559	52
9	.953100	23.32	.998243	.19	.954856	23.51	.045144	51
10	.954499	23.25	.998232	.19	.956267	23.44	.043733	50
11	8.955894	23.17	9.998220	.19	8.957674	23.37	11.042326	49
12	.957284	23.10	.998209	.19	.959075	23.29	.040925	48
13	.958670	23.02	.998197	.19	.960473	23.23	.039527	47
14	.960052	22.95	.998186	.19	.961866	23.14	.038134	46
15	.961429	22.88	.998174	.19	.963255	23.07	.036745	45
16	.962801	22.80	.998163	.19	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.19	.966019	22.93	.033981	43
18	.965534	22.66	.998139	.20	.967394	22.86	.032606	42
19	.966893	22.59	.998128	.20	.968766	22.79	.031234	41
20	.968249	22.52	.998116	.20	.970133	22.71	.029867	40
21	8.969600	22.44	9.998104	.20	8.971496	22.65	11.028504	39
22	.970947	22.38	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.31	.998080	.20	.974209	22.51	.025791	37
24	.973628	22.24	.998068	.20	.975560	22.44	.024440	36
25	.974962	22.17	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.23	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22 10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.04	.016423	30
31	8.982883	21.77	9.997985	.20	8.984899	21.97	11.015101	29
32	.984189	21.70	.997972	.20	.986217	21.91	.013783	28
33	.985491	21.63	.997959	.20	987532	21.84	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.50	.997935	.21	.990149	21.71	.009851	25
36	.989374	21.44	.997922	.21	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.21	.992750	21.58	.007250	23
38	.991943	21.31	.997897	.21	.994045	21.52	.005955	22
39	.993222	21.25	.997885	.21	.995337	21.46	.004663	21
40	.994497	21.19	.997872	.21	.996624	21.40	.003376	20
41	8.995768	21.12	9.997860	.21	8.997908	21.34	11.002092	19
42	.997036	21.06	.997847	.21	.999188	21.27	.000812	18
43	.998299	21.00	.997835	.21	9.000465	21.21	10.999535	17
44	.999560	20.94	.997822	.21	.001738	21.15	.998262	16
45	9.000816	20.87	.997809	.21	.003007	21.09	.996993	15
46	.002069	20.82	.997797	.21	.004272	21.03	.995728	14
47	.003318	20.76	.997784	.21	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.21	.006792	20.91	.993208	12
49	.005805	20.64	.997758	.21	.008047	20.85	.991953	11
50	.007044	20.58	.997745	.21	.009298	20.80	.990702	10
51	9.008278	20.52	9.997732	.21	9.010546	20.74	10.989454	9
52	.009510	20.46	.997719	.21	.011790	20.68	.988210	8
53	.010737	20.40	.997706	.21	.013031	20.62	.986969	7
54	.011962	20.34	.997693	.22	.014268	20.56	.985732	6
55	.013182	20.29	.997680	.22	.015502	20.51	.984498	5
56	.014400	20.23	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.17	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.06	.997628	.22	.020403	20.28	.979597	1
60	.019235	20.00	.997614	.22	.021620	20.23	.978380	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.89	.997588	.22	.024044	20.11	.975956	58
3	.022825	19.84	.997574	.22	.025251	20.06	.974749	57
4	.024016	19.78	.997561	.22	.026455	20.00	.973545	56
5	.025203	19.73	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.67	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.23	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.79	.968763	52
9	.029918	19.51	.997493	.23	.032425	19.74	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.69	.966891	50
11	9.032257	19.41	9.997466	.23	9.034791	19.64	10.965209	49
12	.033421	19.36	.997452	.23	.035969	19.58	.964031	48
13	.034582	19.30	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.38	.959349	44
17	.039197	19.10	.997388	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	18.99	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.94	.997341	.23	.045284	19.18	.954716	40
21	9.043762	18.89	9.997327	.24	9.046434	19.13	10.953566	39
22	.044895	18.84	.997313	.24	.047582	19.08	.952418	38
23	.046026	18.79	.997299	.24	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.24	.049869	18.98	.950131	36
25	.048279	18.70	.997271	.24	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.24	.052144	18.89	.947856	34
27	.050519	18.60	.997242	.24	.053277	18.84	.946723	33
28	.051635	18.55	.997228	.24	.054407	18.79	.945593	32
29	.052749	18.50	.997214	.24	.055535	18.74	.944465	31
30	.053859	18.45	.997199	.24	.056659	18.70	.943341	30
31	9.054966	18.41	9.997185	.24	9.057781	18.65	10.942219	29
32	.056071	18.36	.997170	.24	.058900	18 69	.941100	28
33	.057172	18.31	.997156	.24	.060016	18.55	.939984	27
34	.058271	18.27	.997141	.24	.061130	18.51	.938870	26
35	.059367	18.22	.997127	.24	.062240	18.46	.937760	25
36	.060460	18.17	.997112	.24	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.24	.064453	18.37	.935547	23
38	.062639	18.08	.997083	.25	.065556	18 33	.934444	22
39	.063724	18.04	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.99	.997053	.25	.067752	18.24	.932248	20
41	9.065885	17.94	9.997039	.25	9.068846	18.19	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.86	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.81	.996994	.25	.072113	18.06	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.72	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.68	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.89	.923568	12
49	.074424	17.59	.996919	.25	.077505	17.84	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.76	10.920356	9
52	.077583	17.46	.996874	.25	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.25	.082838	17.63	.917167	6
55	.080719	17.33	.996828	.25	.083891	17.59	.916109	5
56	.081759	17.29	.996812	.26	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.26	.086000	17 51	.914000	3
58	.083832	17.21	.996782	.26	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.26	.088098	17.43	.911902	1
60	.085894	17.13	.996751	.26	.089144	17.38	.910856	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	'
0	9.085894	17.13	9.996751	.26	9.089144	17.38	10.910856	60
1	.086922	17.09	.996735	.26	.090187	17.34	.909813	59
2	.087947	17.04	.996720	.26	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.26	.092266	17.27	.907734	57
4	.089990	16 96	.996688	.26	.093302	17.22	.906698	56
5	.091008	16.92	.996673	.26	.094336	17.19	.905664	55
6	.092024	16.88	.996657	.26	.095367	17.15	.904633	54
7	.093037	16.84	.996641	.26	.096395	17.11	.903605	53
8	.094047	16.80	.996625	.26	.097422	17.07	.902578	52
9	.095056	16.76	.996610	.26	.098446	17.03	.901554	51
10	.096062	16.73	.996594	.26	.099468	16.99	.900532	50
11	9.097065	16 68	9.996578	.27	9.100487	16 95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.91	.898496	48
13	.099065	16.61	.996546	.27	.102519	16.87	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.84	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.49	.996498	.27	.105550	16.76	.894450	44
17	.103037	16.45	.996482	.27	.106556	16.72	.893444	43
18	.104025	16.41	.996465	.27	.107559	16.69	.892441	42
19	.105010	16.38	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.34	.996433	.27	.109559	16.61	.890441	40
21	9.106973	16.30	9.996417	.27	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.54	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.19	.996368	.27	.113533	16.46	.886467	36
25	.110873	16.16	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.27	.115507	16.39	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.36	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.32	.882528	32
29	.114737	16.01	.996285	.28	.118452	16.29	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.94	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.28	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.11	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.07	.875716	25
36	.121417	15.76	.996168	.28	.125249	16.04	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.01	.873789	23
38	.123306	15.69	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.66	.996117	.28	.128130	15.94	.871870	21
40	.125187	15.62	.996100	.28	.129087	15.91	.870913	20
41	9.126125	15.59	9.996083	.29	9.130041	15.87	10.869959	19
42	.127060	15.56	.996066	.29	.130994	15.84	.869006	18
43	.127993	15.52	.996049	.29	.131944	15.81	.868056	17
44	.128925	15.49	.996032	.29	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.29	.133839	15.74	.866161	15
46	.130781	15.42	.995998	.29	.134784	15.71	.865216	14
47	.131706	15.39	.995980	.29	.135726	15.67	.864274	13
48	.132630	15.35	.995963	.29	.136667	15.64	.863333	12
49	.133551	15.32	.995946	.29	.137605	15.61	.862395	11
50	.134470	15.29	.995928	.29	.138542	15.58	.861458	10
51	9.135387	15.25	9.995911	.29	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.29	.140409	15.51	.859591	8
53	.137216	15.19	.995876	.29	.141340	15.48	.858660	7
54	.138128	15.16	.995859	.29	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.29	.143196	15.42	.856804	5
56	.139944	15.09	.995823	.29	.144121	15.39	.855879	4
57	.140850	15.06	.995806	.29	.145044	15.35	.854956	3
58	.141754	15.03	.995788	.29	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.29	.146885	15.29	.853115	1
60	.143555	14.96	.995753	.29	.147803	15.26	.852197	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	
0	9.143555	14.96	9.995753	.30	9.147803	15.26	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.87	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.84	.995681	.30	.151454	15.14	.848546	56
5	.148026	14.81	.995664	.30	.152363	15.11	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.75	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.30	.155077	15.02	.844923	52
9	.151569	14.69	.995591	.30	.155978	14.99	.844022	51
10	.152451	14.66	.995573	.30	.156877	14.96	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.60	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.54	.995501	.31	.160457	14.84	.839543	46
15	.156830	14.51	.995482	.31	.161347	14.81	.838653	45
16	.157700	14.48	.995464	.31	.162236	14.79	.837764	44
17	.158569	14.45	.995446	.31	.163123	14.76	.836877	43
18	.159435	14.42	.995427	.31	.164008	14.73	.835992	42
19	.160301	14.39	.995409	.31	.164892	14.70	.835108	41
20	.161164	14.36	.995390	.31	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.31	9.166654	14.64	10.833346	39
22	.162885	14.30	.995353	.31	.167532	14.61	.832468	38
23	.163743	14.27	.995334	.31	.168409	14.58	.831591	37
24	.164600	14.24	.995316	.31	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.31	.170157	14.53	.829843	35
26	.166307	14.19	.995278	.31	.171029	14.50	.828971	34
27	.167159	14.16	.995260	.31	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.44	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.07	.995203	.32	.174499	14.39	.825501	30
31	9.170547	14.05	9.995184	.32	9.175362	14.36	10.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	13.99	.995146	.32	.177084	14.31	.822916	27
34	.173070	13.96	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.94	.995108	.32	.178799	14.25	.821201	25
36	.174744	13.91	.995089	.32	.179655	14.23	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.86	.995051	.32	.181360	14.17	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.15	.817789	21
40	.178072	13.80	.995013	.32	.183059	14.12	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.09	10.816093	19
42	.179726	13.74	.994974	.32	.184752	14.07	.815248	18
43	.180551	13.72	.994955	.32	.185597	14.04	.814403	17
44	.181374	13.69	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.66	.994916	.33	.187280	13.99	.812720	15
46	.183016	13.64	.994896	.33	.188120	13.96	.811880	14
47	.183834	13.61	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.59	.994857	.33	.189794	13.91	.810206	12
49	.185466	13.56	.994838	.33	.190629	13.89	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.86	.808538	10
51	9.187092	13.51	9.994798	.33	9.192294	13.84	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.81	.806876	8
53	.188712	13.46	.994759	.33	.193953	13.79	.806047	7
54	.189519	13.43	.994739	.33	.194780	13.76	.805220	6
55	.190325	13.41	.994719	.33	.195606	13.74	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.71	.803570	4
57	.191933	13.36	.994680	.33	.197253	13.69	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.66	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.64	.801106	1
60	.194332	13.28	.994620	.33	.199713	13.61	.800287	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.194332	13.28	9.994620	.33	9.199713	13.61	10.800287	60
1	.195129	13.26	.994600	.33	.200529	13.59	.799471	59
2	.195925	13.23	.994580	.33	.201345	13 56	.798655	58
3	.196719	13.21	.994560	.34	.202159	13.54	.797841	57
4	.197511	13 18	.994540	.34	.202971	13.52	.797029	56
5	.198302	13.16	.994519	.34	.203782	13.49	.796218	55
6	.199091	13.13	.994499	.34	.204592	13.47	.795408	54
7	.199879	13.11	.994479	.34	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.34	.206207	13.42	.793793	52
9	.201451	13.06	.994438	.34	.207013	13.40	.792987	51
10	.202234	13.04	.994418	.34	.207817	13.38	.792183	50
11	9.203017	13 01	9.994397	.34	9.208619	13 35	10.791381	49
12	.203797	12.99	.994377	.34	.209420	13.33	.790580	48
13	.204577	12.96	.994357	.34	.210220	13.31	.789780	47
14	.205354	12.94	.994336	.34	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.34	.211815	13.26	.788185	45
16	.206906	12.89	.994295	.34	.212611	13.24	.787389	44
17	.207679	12.87	.994274	.35	.213405	13.21	.786595	43
18	.208452	12.85	.994254	.35	.214198	13.19	.785802	42
19	.209222	12.82	.994233	.35	.214989	13.17	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.15	.784220	40
21	9.210760	12.78	9.994191	.35	9.216568	13.12	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.08	.781858	37
24	.213055	12.71	.994129	.35	.218926	13.05	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.66	.994087	.35	.220492	13.01	.779508	34
27	.215338	12.64	.994066	.35	.221272	12.99	.778728	33
28	.216097	12.61	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.59	.994024	.35	.222830	12.94	.777170	31
30	.217609	12.57	.994003	.35	.223606	12.92	.776394	30
31	9.218363	12.55	9.993981	.35	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	225929	12.86	.774071	27
34	.220618	12 48	.993918	.35	.226700	12.84	.773300	26
35	.221367	12.46	.993896	.36	.227471	12.81	.772529	25
36	.222115	12.44	.993875	.36	.228239	12.79	.771761	24
37	.222861	12.42	.993854	.36	.229007	12.77	.770993	23
38	.223606	12.39	.993832	.36	.229773	12.75	.770227	22
39	.224349	12.37	.993811	.36	.230539	12.73	.769461	21
40	.225092	12.35	.993789	.36	.231302	12.71	.768698	20
41	9.225833	12.33	9.993768	.36	9.232065	12.69	10.767935	19
42	.226573	12.31	.993746	.36	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.36	.233586	12.65	.766414	17
44	.228048	12.26	.993703	.36	.234345	12.62	.765655	16
45	.228784	12.24	.993681	.36	.235103	12.60	.764897	15
46	.229518	12.22	.993660	.36	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.36	.236614	12.56	.763386	13
48	.230984	12.18	.993616	.36	.237368	12.54	.762632	12
49	.231714	12.16	.993594	.37	.238120	12.52	.761880	11
50	.232444	12.14	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.09	.993528	.37	.240371	12.46	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.44	.758882	7
54	.235349	12.05	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.01	.993440	.37	.243354	12.38	.756646	4
57	.237515	11.99	.993418	.37	.244097	12.36	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.34	.755161	2
59	.238953	11.95	.993374	.37	.245579	12.32	.754421	1
60	.239670	11.93	.993351	.37	.246319	12.30	.753681	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240386	11.91	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.89	.993307	.37	.247794	12.26	.752206	58
3	.241814	11.87	.993285	.37	.248530	12.24	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.22	.750736	56
5	.243237	11.83	.993240	.37	.249998	12.20	.750002	55
6	.243947	11.81	.993217	.38	.250730	12.18	.749270	54
7	.244656	11.79	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.75	.993149	.38	.252920	12.13	.747080	51
10	.246775	11.73	.993127	.38	.253648	12.11	.746352	50
11	9.247478	11.71	9.993104	.38	9.254374	12.09	10.745626	49
12	.248181	11.69	.993081	.38	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.01	.742731	45
16	.250980	11.61	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.59	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.58	.992944	.38	.259429	11.96	.740571	42
19	.253067	11.56	.992921	.38	.260146	11.94	.739854	41
20	.253761	11.54	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.89	.737708	38
23	.255834	11.48	.992829	.39	.263005	11.87	.736995	37
24	.256523	11.46	.992806	.39	.263717	11.85	.736283	36
25	.257211	11.44	.992783	.39	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.39	.265138	11.81	.734862	34
27	.258583	11.41	.992736	.39	.265847	11.79	.734153	33
28	.259268	11.39	.992713	.39	.266555	11.78	.733445	32
29	.259951	11.37	.992690	.39	.267261	11.76	.732739	31
30	.260633	11.35	.992666	.39	.267967	11.74	.732033	30
31	9.261314	11.33	9.992643	.39	9.268671	11.72	10.731329	29
32	.261994	11.31	.992619	.39	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.39	.270077	11.69	.729923	27
34	.263351	11.28	.992572	.39	.270779	11.67	.729221	26
35	.264027	11.26	.992549	.39	.271479	11.65	.728521	25
36	.264703	11.24	.992525	.39	.272178	11.64	.727822	24
37	.265377	11.22	.992501	.39	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.19	.992454	.40	.274269	11.58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11 57	.725036	20
41	9.268065	11.15	9.992406	.40	9.275658	11.55	10 724342	19
42	.268734	11.13	.992382	.40	.276351	11.53	.723649	18
43	.269402	11.11	.992359	.40	.277043	11.51	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.06	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.05	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.40	.280488	11.43	.719512	12
49	.273388	11.01	.992214	.40	.281174	11.41	.718826	11
50	.274049	10.99	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.96	.992142	.40	.283225	11.36	.716775	8
53	.276024	10.94	.992117	.41	.283907	11.35	.716093	7
54	.276681	10.92	.992093	.41	.284588	11.33	.715412	6
55	.277337	10.91	.992069	.41	.285268	11.31	.714732	5
56	.277991	10.89	.992044	.41	.285947	11 30	.714053	4
57	.278644	10.87	.992020	.41	.286624	11.28	.713376	3
58	.279297	10.86	.991996	.41	.287301	11.26	.712699	2
59	.279948	10.84	.991971	.41	.287977	11.25	.712023	1
60	.280599	10.82	.991947	.41	.288652	11.23	.711348	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.280599	10.82	9.991947	.41	9.288652	11.23	10.711348	60
1	.281248	10.81	.991922	.41	.289326	11.22	.710674	59
2	.281897	10.79	.991897	.41	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.41	.290671	11.18	.709329	57
4	.283190	10 76	.991848	.41	.291342	11.17	.708658	56
5	.283836	10.74	.991823	.41	.292013	11.15	.707987	55
6	.284480	10.72	.991799	.41	.292682	11.14	.707318	54
7	.285124	10.71	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.69	.991749	.42	.294017	11.11	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.09	.705316	51
10	.287048	10.66	.991699	.42	.295349	11.07	.704651	50
11	9.287687	10.64	9.991674	.42	9.296013	11.06	10.703987	49
12	.288326	10.63	.991649	.42	.296677	11.04	.703323	48
13	.288964	10.61	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.59	.991599	.42	.298001	11.01	.701999	46
15	.290236	10.58	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.56	.991549	.42	.299322	10.98	.700678	44
17	.291504	10.54	.991524	.42	.299980	10.96	.700020	43
18	.292137	10.53	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.51	.991473	.42	.301295	10.93	.698705	41
20	.293399	10.50	.991448	.42	.301951	10.92	.698049	40
21	9.294029	10.48	9.991422	.42	9.302607	10.90	10.697393	39
22	.294658	10.46	.991397	.42	.303261	10.89	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.87	.696086	37
24	.295913	10.43	.991346	.43	.304567	10.86	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.84	.694782	35
26	.297164	10.40	.991295	.43	.305869	10.83	.694131	34
27	.297788	10.39	.991270	.43	.306519	10.81	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.36	.991218	.43	.307815	10.78	.692185	31
30	.299655	10.34	.991193	.43	.308463	10 77	.691537	30
31	9.300276	10.32	9.991167	.43	9.309109	10.75	10.690891	29
32	.300895	10.31	.991141	.43	.309754	10.74	.690246	28
33	.301514	10.29	.991115	.43	.310398	10.73	.689602	27
34	.302132	10.28	.991090	.43	.311042	10.71	.688958	26
35	.302748	10.26	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312967	10.67	.687033	23
38	.304593	10.22	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.64	.685753	21
40	.305819	10.19	.990934	.44	.314885	10.62	.685115	20
41	9.306430	10.17	9.990908	.44	9.315523	10.61	10.684477	19
42	.307041	10.16	.990882	.44	.316159	10.60	.683841	18
43	.307650	10.14	.990855	.44	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.44	.317430	10.57	.682570	16
45	.308867	10.11	.990803	.44	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.44	.318697	10.54	.681303	14
47	.310080	10.08	.990750	.44	.319329	10.53	.680671	13
48	.310685	10.07	.990724	.44	.319961	10.51	.680039	12
49	.311289	10.05	.990697	.44	.320592	10.50	.679408	11
50	.311893	10.04	.990671	.44	.321222	10.48	.678778	10
51	9.312495	10.03	9.990644	.44	9.321851	10.47	10.678149	9
52	.313097	10.01	.990618	.44	.322479	10.45	.677521	8
53	.313698	10.00	.990591	.44	.323106	10.44	.676894	7
54	.314297	9.98	.990565	.44	.323733	10.43	.676267	6
55	.314897	9.97	.990538	.44	.324358	10.41	.675642	5
56	.315495	9.96	.990511	.45	.324983	10.40	.675017	4
57	.316092	9.94	.990485	.45	.325607	10.39	.674393	3
58	.316689	9.93	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.91	.990431	.45	.326853	10.36	.673147	1
60	.317879	9.90	.990404	.45	.327475	10.35	.672525	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.317879	9.90	9.990404	.45	9.327474	10.35	10.672526	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.86	.990324	.45	.329334	10.30	.670666	57
4	.320249	9.84	.990297	.45	.329953	10.29	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.45	.331187	10.26	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.79	.990188	.45	.332418	10.24	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.23	.666967	51
10	.323780	9.76	.990134	.45	.333646	10.21	.666354	50
11	9.324366	9.75	9.990107	.46	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.46	.334871	10.19	.665129	48
13	.325534	9.72	.990052	.46	.335482	10.17	.664518	47
14	.326117	9.70	.990025	.46	.336093	10.16	.663907	46
15	.326700	9.69	.989997	.46	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.46	.337311	10.13	.662689	44
17	.327862	9.66	.989942	.46	.337919	10.12	.662081	43
18	.328442	9.65	.989915	.46	.338527	10.11	.661473	42
19	.329021	9.64	.989887	.46	.339133	10 10	.660867	41
20	.329599	9.62	.989860	.46	.339739	10.08	.660261	40
21	9.330176	9.61	9.989832	.46	9.340344	10.07	10.659656	39
22	.330753	9.60	.989804	.46	.340948	10.06	.659052	38
23	.331329	9.58	.989777	.46	.341552	10.04	.658448	37
24	.331903	9.57	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.56	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.54	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.53	.989665	.47	.343958	9.99	.656042	33
28	.334195	9.52	.989637	.47	.344558	9.98	.655442	32
29	.334766	9.50	.989609	.47	.345157	9.97	.654843	31
30	.335337	9.49	.989582	.47	.345755	9.96	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.94	10.653647	29
32	.336475	9.46	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.92	.652455	27
34	.337610	9.44	.989469	.47	.348141	9.91	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.41	.989413	.47	.349329	9.88	.650671	24
37	.339306	9.40	.989384	.47	.349922	9.87	.650078	23
38	.339871	9.39	.989356	.47	.350514	9.86	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.36	.989300	.47	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.34	.989243	.47	.352876	9.81	.647124	18
43	.342679	9.32	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.31	.989186	.47	.354053	9.79	.645947	16
45	.343797	9.30	.989157	.47	.354640	9.77	.645360	15
46	.344355	9.29	.989128	.48	.355227	9.76	.644773	14
47	.344912	9.27	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.26	.989071	.48	.356398	9.74	.643602	12
49	.346024	9.25	.989042	.48	.356982	9.73	.643018	11
50	.346579	9.24	.989014	.48	.357566	9.71	.642434	10
51	9.347134	9.22	9.988985	.48	9 358149	9.70	10.641851	9
52	.347687	9.21	.988956	.48	.358731	9.69	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.68	.640687	7
54	.348792	9.19	.988898	.48	.359893	9.67	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.66	.639526	5
56	.349893	9.16	.988840	.48	.361053	9 65	.638947	4
57	.350443	9.15	.988811	.49	.361632	9.63	.638368	3
58	.350992	9.14	.988782	.49	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.49	.362787	9.61	.637213	1
60	.352088	9.11	.988724	.49	363364	9.60	.636636	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	.352635	9 10	.988695	.49	.363940	9.59	.636060	59
2	.353181	9.09	.988666	.49	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.49	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.49	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.49	.366237	9.54	.633763	55
6	.355358	9.04	.988548	.49	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.49	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.49	.367953	9.51	.632047	52
9	.356984	9.01	.988460	.49	.368524	9.50	.631476	51
10	.357524	8.99	.988430	.49	.369094	9.49	.630906	50
11	9.358064	8.98	9.988401	.49	9.369663	9.48	10.630337	49
12	.358603	8.97	.988371	.49	.370232	9.46	.629768	48
13	.359141	8.96	.988342	.49	.370799	9.45	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.44	.628633	46
15	.360215	8.93	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.50	.372499	9.42	.627501	44
17	.361287	8.91	.988223	.50	.373064	9.41	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.89	.988163	.50	.374193	9.39	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	.363954	8.86	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.84	.988043	.50	.376442	9.34	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.81	.987953	.50	.378122	9.31	.621878	34
27	.366604	8.80	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.79	.987892	.50	.379239	9.29	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.76	.987832	.51	.380354	9.27	.619646	30
31	9.368711	8.75	9.987801	.51	9.380910	9.26	10.619090	29
32	.369236	8.74	.987771	.51	.381466	9.25	.618534	28
33	.369761	8.73	.987740	.51	.382020	9.24	.617980	27
34	.370285	8.72	.987710	.51	.382575	9.23	.617425	26
35	.370808	8.71	.987679	.51	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.51	.383682	9.21	.616318	24
37	.371852	8.69	.987618	.51	.384234	9.20	.615766	23
38	.372373	8.67	.987588	.51	.384786	9.19	.615214	22
39	.372894	8.66	.987557	.51	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.51	.385888	9.17	.614112	20
41	9.373933	8.64	9.987496	.51	9.386438	9.15	10.613562	19
42	.374452	8.63	.987465	.51	.386987	9.14	.613013	18
43	.374970	8.62	.987434	.51	.387536	9.13	.612464	17
44	.375487	8.61	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.11	.611369	15
46	.376519	8.59	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.58	.987310	.52	.389724	9.09	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.56	.987248	.52	.390815	9.07	.609185	11
50	.378577	8.54	.987217	.52	.391360	9.06	.608640	10
51	9.379089	8.53	9.987186	.52	9.391903	9.05	10.608097	9
52	379601	8.52	.987155	.52	.392447	9.04	.607553	8
53	.380113	8.51	.987124	.52	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.02	.606469	6
55	.381134	8.49	.987061	.52	.394073	9.01	.605927	5
56	.381643	8.48	.987030	.52	.394614	9.00	.605386	4
57	.382152	8.47	.986998	.52	.395154	8.99	.604846	3
58	.382661	8.46	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.52	.396233	8.97	.603767	1
60	.383675	8.44	.986904	.52	.396771	8.96	.603229	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.383675	8.44	9.986904	.52	9.396771	8.96	10.603229	60
1	.384182	8.43	.986873	.53	.397309	8.96	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.41	.986809	.53	.398383	8.94	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.39	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.53	.399990	8.91	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.36	.986651	.53	.401058	8.89	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.34	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.86	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.31	.986491	.53	.403718	8.84	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.83	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.82	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.81	.594692	44
17	.392199	8.26	.986363	.54	.405836	8.80	.594164	43
18	.392695	8.25	.986331	.54	.406364	8.79	.593636	42
19	.393191	8.24	.986299	.54	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.54	.407419	8.77	.592581	40
21	9.394179	8.22	9.986234	.54	9.407945	8.76	10 592055	39
22	.394673	8.21	.986202	.54	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.54	.408997	8.74	.591003	37
24	.395658	8.19	.986137	.54	.409521	8.74	.590479	36
25	.396150	8.18	.986104	.54	.410045	8.73	.589955	35
26	.396641	8.17	.986072	.54	.410569	8.72	.589431	34
27	.397132	8.17	.986039	.54	.411092	8.71	.588908	33
28	.397621	8.16	.986007	.54	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.54	.412137	8.69	.587863	31
30	.398600	8.14	.985942	.54	.412658	8.68	.587342	30
31	9.399088	8.13	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.66	.586301	28
33	.400062	8.11	.985843	.55	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.64	.585262	26
35	.401035	8.09	.985778	.55	.415257	8.64	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.06	.985679	.55	.416810	8.61	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.04	.985613	.55	.417842	8.59	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.01	.985514	.55	.419387	8.56	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.55	.580099	16
45	.405862	7.99	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.56	.420927	8.54	.579073	14
47	.406820	7.97	.985380	.56	.421440	8.53	.578560	13
48	.407299	7.96	.985347	.56	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.56	.422463	8.51	.577537	11
50	.408254	7.94	.985280	.56	.422974	8.50	.577026	10
51	9.408731	7.94	9.985247	.56	9.423484	8.49	10.576516	9
52	.409207	7.93	.985213	.56	.423993	8.48	.576007	8
53	.409682	7.92	.985180	.56	.424503	8.48	.575497	7
54	.410157	7.91	.985146	.56	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.56	.425519	8.46	.574481	5
56	.411106	7.89	.985079	.56	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.56	.426534	8.44	.573466	3
58	.412052	7.87	.985011	.56	.427041	8.43	.572959	2
59	.412524	7.86	.984978	.56	.427547	8.43	.572453	1
60	.412996	7.85	.984944	.56	.428052	8.42	.571948	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.412996	7.85	9.984944	.57	9.428052	8.42	10.571948	60
1	.413467	7.84	.984910	.57	.428557	8.41	.571443	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.39	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.81	.984774	.57	.430573	8.38	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.79	.984706	.57	.431577	8.36	.568423	53
8	.416751	7.78	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.77	.984637	.57	.432580	8.34	.567420	51
10	.417684	7.76	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.32	10.566420	49
12	.418615	7.74	.984535	.57	.434080	8.32	.565920	48
13	.419079	7.73	.984500	.57	.434579	8.31	.565421	47
14	.419544	7.73	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.29	.564424	45
16	.420470	7.71	.984397	.58	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.69	.984328	.58	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.26	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.58	9.438554	8.24	10.561446	39
22	.423238	7.66	.984190	.58	.439048	8.23	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.23	.560457	37
24	.424156	7.64	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.21	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.61	.984015	.58	.441514	8.19	.558486	33
28	.425987	7.60	.983981	.58	.442006	8.19	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.59	.983911	.58	.442988	8.17	.557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.16	10.556521	29
32	.427809	7.57	.983840	.59	.443968	8.16	.556032	28
33	.428263	7.56	.983805	.59	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.59	.444947	8.14	.555053	26
35	.429170	7.54	.983735	.59	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.59	.445923	8.12	.554077	24
37	.430075	7.52	.983664	.59	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.59	.446898	8.11	.553102	22
39	.430978	7.51	.983594	.59	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.59	.447870	8.09	.552130	20
41	9.431879	7.49	9.983523	.59	9.448356	8.09	10.551644	19
42	.432329	7.49	.983487	.59	.448841	8.08	.551159	18
43	.432778	7.48	.983452	.59	.449326	8.07	.550674	17
44	.433226	7.47	.983416	.59	.449810	8.06	.550190	16
45	.433675	7.46	.983381	.59	.450294	8.06	.549706	15
46	.434122	7.45	.983345	.59	.450777	8.05	.549223	14
47	.434569	7.44	.983309	.59	.451260	8.04	.548740	13
48	.435016	7.44	.983273	.60	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	.435908	7.42	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.41	9.983166	.60	9.453187	8.01	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	7.99	.545852	7
54	.437686	7.39	.983058	.60	.454628	7.99	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.36	.982950	.60	.456064	7.96	.543936	3
58	.439456	7.36	.982914	.60	.456542	7.96	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	.440338	7.34	.982842	.60	.457496	7.94	.542504	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.440338	7.34	9.982842	.60	9.457496	7.94	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.32	.982769	.61	.458449	7.93	.541551	58
3	.441658	7.31	.982733	.61	.458925	7.92	.541075	57
4	.442096	7.31	.982696	.61	.459400	7.91	.540600	56
5	.442535	7.30	.982660	.61	.459875	7.90	.540125	55
6	.442973	7.29	.982624	.61	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.61	.460823	7.89	.539177	53
8	.443847	7.27	.982551	.61	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.61	.461770	7.88	.538230	51
10	.444720	7.26	.982477	.61	.462242	7.87	.537758	50
11	9.445155	7.25	9.982441	.61	9.462714	7.86	10.537286	49
12	.445590	7.24	.982404	.61	.463186	7.85	.536814	48
13	.446025	7.23	.982367	.61	.463658	7.85	.536342	47
14	.446459	7.28	.982331	.61	.464129	7.84	.535871	46
15	.446893	7.22	.982294	.61	.464599	7.83	.535401	45
16	.447326	7.21	.982257	.61	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.81	.533992	42
19	.448623	7.19	.982146	.62	.466476	7.80	.533524	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.79	10.532587	39
22	.449915	7.16	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.16	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.14	.981924	.62	.469280	7.76	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.62	.470676	7.74	.529324	32
29	.452915	7.11	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981699	.63	9.472068	7.72	10.527932	29
32	.454194	7.09	.981662	.63	.472532	7.71	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.71	.527005	27
34	.455044	7.07	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.63	.473919	7.69	.526081	25
36	.455893	7.06	.981512	.63	.474381	7.69	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.04	.981436	.63	.475303	7.67	.524697	22
39	.457162	7.04	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.66	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.01	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.01	.981247	.63	.477601	7.64	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	6.99	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.64	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.64	.479432	7.61	.520568	13
48	.460946	6.97	.981057	.94	.479889	7.61	.520111	12
49	.461364	6.96	.981019	.64	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.64	.480801	7.59	.519199	10
51	9.462199	6.95	9.980942	.64	9.481257	7.59	10.518743	9
52	.462616	6.94	.980904	.64	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.64	.482167	7.57	.517833	7
54	.493448	6.93	.980827	.64	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.64	.483075	7.56	.516925	5
56	.464279	6.91	.980750	.64	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.64	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.64	.484435	7.54	.515565	2
59	.465522	6.89	.980635	.64	.484887	7.53	.515113	1
60	.465935	6.88	.980596	.64	.485339	7.53	.514661	0
	Cosine.	D.	Sine.		Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.465935	6.88	9.980596	.64	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.64	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.51	.513758	58
3	.467173	6.86	.980480	.65	.486693	7.51	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.49	.512407	55
6	.468407	6.84	.980364	.65	.488043	7.49	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.47	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.81	.980208	.65	.489838	7.46	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.46	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.79	.980091	.65	.491180	7.44	.508820	47
14	.471679	6.78	.980052	.65	.491627	7.44	.508373	46
15	.472086	6.78	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.76	.979934	.66	.492965	7.42	.507035	43
18	.473304	6.76	.979895	.66	.493410	7.41	.506590	42
19	.473710	6.75	.979855	.66	.493854	7.40	.506146	41
20	.474115	6.74	.979816	.66	.494299	7.40	.505701	40
21	9.474519	6.74	9.979776	.66	9.494743	7.40	10.505257	39
22	.474923	6.73	.979737	.66	.495186	7.39	.504814	38
23	.475327	6.72	.979697	.66	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.66	.496073	7.37	.503927	36
25	.476133	6.71	.979618	.66	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.66	.496957	7.36	.503043	34
27	.476938	6.69	.979539	.66	.497399	7.36	.502601	33
28	.477340	6.69	.979499	.66	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.66	.498282	7.34	.501718	31
30	.478142	6.67	.979420	.66	.498722	7.34	.501278	30
31	9.478542	6.67	9.979380	.66	9.499163	7.33	10.500837	29
32	.478942	6.66	.979340	.66	.499603	7.33	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.31	.499519	26
35	.480140	6.64	.979220	.67	.500920	7.31	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.63	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.67	.502235	7.29	.497765	22
39	.481731	6.61	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.61	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.59	.978939	.67	.503982	7.27	.496018	18
43	.483316	6.59	.978898	.67	.504418	7.26	.495582	17
44	.483712	6.58	.978858	.67	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.24	.494276	14
47	.484895	6.56	.978736	.67	.506159	7.24	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.68	.507027	7.22	.492973	11
50	.486075	6.54	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.53	9.978574	.68	9.507893	7.21	10.492107	9
52	.486860	6.53	.978533	.68	.508326	7.21	.491674	8
53	.487251	6.52	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.51	.978452	.68	.509191	7.19	.490809	6
55	.488034	6.51	.978411	.68	.509622	7.19	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.49	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.16	.488654	1
60	.489982	6.48	.978206	.68	.511776	7.16	.488224	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

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M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.489982	6.48	9.978206	.68	9.511776	7.16	10.488224	60
1	.490371	6.48	.978165	.68	.512206	7.16	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.46	.978083	.69	.513064	7.14	.486936	57
4	.491535	6.46	.978042	.69	.513493	7.14	.486507	56
5	.491922	6.45	.978001	.69	.513921	7.13	.486079	55
6	.492308	6.44	.977959	.69	.514349	7.13	.485651	54
7	.492695	6.44	.977918	.69	.514777	7.12	.485223	53
8	.493081	6.43	.977877	.69	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.69	.515631	7.11	.484369	51
10	.493851	6.42	.977794	.69	.516057	7.10	.483943	50
11	9.494236	6.41	9.977752	.69	9.516484	7.10	10.483516	49
12	.494621	6.41	.977711	.69	.516910	7.09	.483090	48
13	.495005	6.40	.977669	.69	.517335	7.09	.482665	47
14	.495388	6.39	.977628	.69	.517761	7.08	.482239	46
15	.495772	6.39	.977586	.69	.518185	7.08	.481815	45
16	.496154	6.38	.977544	.70	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.06	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.06	.480542	42
19	.497301	6.36	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.36	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.35	9.977335	.70	9.520728	7.04	10.479272	39
22	.498444	6.34	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.34	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.31	.977083	.70	.523259	7.01	.476741	33
28	.500721	6.31	.977041	.70	.523680	7.01	.476320	32
29	.501099	6.30	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.29	.976957	.70	.524520	6.99	.475480	30
31	9.501854	6.29	9.976914	.70	9.524939	6.99	10.475061	29
32	.502231	6.28	.976872	.71	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.71	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.71	.526197	6 97	.473803	26
35	.503360	6.26	.976745	.71	.526615	6.97	.473385	25
36	.503735	6.26	.976702	.71	.527033	6.96	.472967	24
37	.504110	6.25	.976660	.71	.527451	6.96	.472549	23
38	.504485	6.25	.976617	.71	.527868	6.95	.472132	22
39	.504860	6.24	.976574	.71	.528285	6.95	.471715	21
40	.505234	6 23	.976532	.71	.528702	6.94	.471298	20
41	9.505608	6.23	9.976489	.71	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.71	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.71	.529950	6.93	.470050	17
44	.506727	6.21	.976361	.71	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.71	.530781	6.91	.469219	15
46	.507471	6.20	.976275	.71	.531196	6.91	.468804	14
47	.507843	6.19	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.19	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.89	.467561	11
50	.508956	6.18	.976103	.72	.532853	6.89	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.16	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.16	.975974	.72	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.86	.465084	5
56	.511172	6.14	.975844	.72	.535328	6.86	.464672	4
57	.511540	6.13	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.72	.536561	6.84	.463439	1
60	.512642	6.12	.975670	.72	.536972	6.84	.463028	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.512642	6.12	9.975670	.73	9.536972	6.84	10.463028	60
1	.513009	6.11	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.11	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.73	.538202	6.82	.461798	57
4	.514107	6.09	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.09	.975452	.73	.539020	6.81	.460980	55
6	.514837	6.08	.975408	.73	.539429	6.81	.460571	54
7	.515202	6.08	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.79	.459347	51
10	.516294	6.06	.975233	.73	.541061	6.79	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.05	.975145	.73	.541875	6.78	.458125	48
13	.517382	6.04	.975101	.73	.542281	6.77	.457719	47
14	.517745	6.04	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.03	.975013	.73	.543094	6.76	456906	45
16	.518468	6.03	.974969	.74	.543499	6.76	.456501	44
17	.518829	6.02	.974925	.74	.543905	6.75	.456095	43
18	.519190	6.01	.974880	.74	.544310	6.75	.455690	42
19	.519551	6.01	.974836	.74	.544715	6.74	.455285	41
20	.519911	6.00	.974792	.74	.545119	6.74	.454881	40
21	9.520271	6.00	9.974748	.74	9.545524	6.73	10.454476	39
22	.520631	5.99	.974703	.74	.545928	6.73	.454072	38
23	.520990	5.99	.974659	.74	.546331	6.72	.453669	37
24	.521349	5.98	.974614	.74	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.74	.547138	6.71	.452862	35
26	.522066	5.97	.974525	.74	.547540	6.71	.452460	34
27	.522424	5.96	.974481	.74	.547943	6.70	.452057	33
28	.522781	5.96	.974436	.74	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.74	.548747	6.69	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.69	.450851	30
31	9.523852	5.94	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.94	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.93	.974167	.75	.550752	6.67	.449248	26
35	.525275	5.92	.974122	.75	.551152	6.66	.448848	25
36	.525630	5.91	.974077	.75	.551552	6.66	.448448	24
37	.525984	5.91	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.90	.973942	.75	.552750	6.65	.447250	21
40	.527046	5.89	.973897	.75	.553149	6.64	.446851	20
41	9.527400	5.89	9.973852	.75	9.553548	6.64	10.446452	19
42	.527753	5.88	.973807	.75	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.63	.445656	17
44	.528458	5.87	.973716	.76	.554741	6.62	.445259	16
45	.528810	5.87	.973671	.76	.555139	6.62	.444861	15
46	.529161	5.86	.973625	.76	.555536	6.61	.444464	14
47	.529513	5.86	.973580	.76	.555933	6.61	.444067	13
48	.529864	5.85	.973535	.76	.556329	6.60	.443671	12
49	.530215	5.85	.973489	.76	.556725	6.60	.443275	11
50	.530565	5.84	.973444	.76	.557121	6.59	.442879	10
51	9.530915	5.84	9.973398	.76	9.557517	6.59	10.442483	9
52	.531265	5.83	.973352	.76	.557913	6.59	.442087	8
53	.531614	5.82	.973307	.76	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.76	.558702	6.58	.441298	6
55	.532312	5.81	.973215	.76	.559097	6.57	.440903	5
56	.532661	5.81	.973169	.76	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.76	.559885	6.56	.440115	3
58	.533357	5.80	.973078	.76	.560279	6.56	.439721	2
59	.533704	5.79	.973032	.77	.560673	6.55	.439327	1
60	.534052	5.78	.972986	.77	.561066	6.55	.438934	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.534052	5.78	9,972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.77	.972940	.77	.561459	6.54	.438541	59
2	.534745	5.77	.972894	.77	.561851	6.54	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.76	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.76	.972755	.77	.563028	6.53	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.74	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.74	.972617	.77	.564202	6.51	.435798	52
9	.537163	5.73	.972570	.77	.564592	6.51	.435408	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.77	9 565373	6.50	10.434627	49
12	.538194	5.72	.972431	.78	.565763	6.49	.434237	48
13	.538538	5.71	.972385	.78	.566153	6.49	.433847	47
14	.538880	5.71	.972338	.78	.566542	6.49	.433458	46
15	.539223	5.70	.972291	.78	.566932	6.48	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.69	.972198	.78	.567709	6.47	.432291	43
18	.540249	5.69	.972151	.78	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6 46	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.46	.431127	40
21	9.541272	5.67	9.972011	.78	9.569261	6 45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.66	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.66	.971870	.78	.570422	6.44	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.44	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.64	.971729	.79	.571581	6 43	.428419	33
28	.543649	5.64	.971682	.79	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.79	.572352	6.42	.427648	31
30	.544325	5.63	.971588	.79	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.79	9.573123	6.41	10.426877	29
32	.545000	5.62	.971493	.79	.573507	6.41	.426493	28
33	.545338	5.61	.971446	.79	.573892	6.40	.426108	27
34	.545674	5.61	.971398	.79	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.79	.574660	6.39	.425340	25
36	.546347	5.60	.971303	.79	.575044	6.39	.424956	24
37	.546683	5.59	.971256	.79	.575427	6.39	.424573	23
38	.547019	5.59	.971208	.79	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.79	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.79	.576576	6.37	.423424	20
41	9.548024	5.57	9.971066	.80	9.576958	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.36	.422659	18
43	.548693	5.56	.970970	.80	.577723	6.36	.422277	17
44	.549027	5.56	.970922	.80	.578104	6.36	.421896	16
45	.549360	5.55	.970874	.80	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.54	.970779	.80	.579248	6.34	.420752	13
48	.550359	5.54	.970731	.80	.579629	6.34	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.34	.419991	11
50	.551024	5.53	.970635	.80	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.51	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.51	.970394	.80	.582286	6.31	.417714	5
56	.553010	5.50	.970345	.81	.582665	6.31	.417335	4
57	.553341	5.50	.970297	.81	.583043	6 30	.416957	3
58	.553670	5.49	.970249	.81	.583422	6.30	.416578	2
59	.554000	5.49	.970200	.81	.583800	6.29	.416200	1
60	.554329	5.48	.970152	.81	.584177	6.29	.415823	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.554329	5.48	9.970152	.81	9.584177	6.29	10.415823	60
1	.554658	5.48	.970103	.81	.584555	6.29	.415445	59
2	.554987	5.47	.970055	.81	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.81	.585309	6.28	.414691	57
4	.555643	5.46	.969957	.81	.585686	6.27	.414314	56
5	.555971	5.46	.969909	.81	.586062	6.27	.413938	55
6	.556299	5.45	.969860	.81	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.81	.586815	6.26	.413185	53
8	.556953	5.44	.969762	.81	.587190	6.26	.412810	52
9	.557280	5.44	.969714	.81	.587566	6 25	.412434	51
10	.557606	5.43	.969665	.81	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.43	.969567	.82	.588691	6.24	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.24	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.41	.969420	.82	.589814	6.23	.410186	45
16	.559558	5.41	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.39	.969223	.82	.591308	6.22	.408692	41
20	.560855	5.39	.969173	.82	.591681	6.21	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.21	10.407946	39
22	.561501	5.38	.969075	.82	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592798	6.20	.407202	37
24	.562146	5 37	.968976	.82	.593170	6.19	.406829	36
25	.562468	5.36	.968926	.83	.593542	6.19	.406458	35
26	.562790	5.36	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.36	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.35	.968777	.83	.594656	6.18	.405344	32
29	.563755	5.35	.968728	.83	.595027	6.17	.404973	31
30	.564075	5.34	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.34	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.16	.403862	28
33	.565036	5 33	.968528	.83	.596508	6.16	.403492	27
34	.565356	5.32	.968479	.83	.596878	6.16	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.31	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.31	.968329	.83	.597985	6.15	.402015	23
38	.566632	5.31	.968278	.83	.598354	6.14	.401646	22
39	.566951	5 30	.968228	.84	.598722	6.14	.401278	21
40	.567269	5.30	.968178	.84	.599091	6.13	.400909	20
41	9.567587	5.29	9.968128	.84	9.599459	6.13	10.400541	19
42	.567904	5.29	.968078	.84	.599827	6.13	.400173	18
43	.568222	5.28	.968027	.84	.600194	6.12	.399806	17
44	.568539	5.28	.967977	.84	.600562	6.12	.399438	16
45	.568856	5.28	.967927	.84	.600929	6.11	.399071	15
46	.569172	5.27	.967876	.84	.601296	6.11	.398704	14
47	.569488	5.27	.967826	.84	.601662	6.11	.398338	13
48	.569804	5.26	.967775	.84	.602029	6.10	.397971	12
49	.570120	5.26	.967725	.84	.602395	6.10	.397605	11
50	.570435	5.25	.967674	.84	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.84	9.603127	6.09	10.396873	9
52	.571066	5.24	.967573	.84	.603493	6.09	.396507	8
53	.571380	5.24	.967522	.85	.603858	6.09	.396142	7
54	.571695	5.23	.967471	.85	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.23	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.22	.967319	.85	.605317	6.07	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.21	.967217	.85	.606046	6.06	.393954	1
60	.573575	5.21	.967166	.85	.606410	6.06	.393590	0
	Cosine	D.	Sine.	D.	Cotang.	D.	Tang.	M

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M	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	
0	9.573575	5.21	9 967166	.85	9.606410	6.06	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.06	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.19	.967013	.85	.607500	6.05	.392500	57
4	.574824	5.19	.966961	.85	.607863	6.04	.392137	56
5	.575136	5.19	.966910	.85	.608225	6.04	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.04	.391412	54
7	.575758	5.18	.966808	.85	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.86	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.86	.609674	6.03	.390326	51
10	.576689	5.16	.966653	.86	.610036	6.02	.389964	50
11	9.576999	5.16	9.966602	.86	9.610397	6.02	10.389603	49
12	.577309	5.16	.966550	.86	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.86	.611120	6.01	.388880	47
14	.577927	5.15	.966447	.86	.611480	6.01	.388520	46
15	.578236	5.14	.966395	.86	.611841	6.01	.388159	45
16	.578545	5.14	.966344	.86	.612201	6.00	.387799	44
17	.578853	5.13	.966292	.86	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.86	.612921	6.00	.387079	42
19	.579470	5.13	.966188	.86	.613281	5.99	.386719	41
20	.579777	5.12	.966136	.86	.613641	5.99	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	.580392	5.11	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.11	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.11	.965928	.87	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.09	.965772	.87	.616151	5.96	.383849	33
28	.582229	5.09	.965720	.87	.616509	5.96	.383491	32
29	.582535	5.09	.965668	.87	.616867	5.96	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.95	.382776	30
31	9.583145	5.08	9.965563	.87	9.617582	5.95	10.382418	29
32	.583449	5.07	.965511	.87	.617939	5.95	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.94	.381705	27
34	.584058	5.06	.965406	.87	.618652	5.94	.381348	26
35	.584361	5.06	.965353	.88	.619008	5.94	.380992	25
36	.584665	5.06	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.05	.965248	.88	.619721	5.93	.380279	23
38	.585272	5.05	.965195	.88	.620076	5.93	.379924	22
39	.585574	5.04	.965143	.88	.620432	5.92	.379568	21
40	.585877	5.04	.965090	.88	.620787	5 92	.379213	20
41	9 586179	5.03	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.03	.964984	.88	.621497	5.91	.378503	18
43	.586783	5.03	.964931	.88	.621852	5.91	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.02	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.01	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.01	.964719	.88	.623269	5.89	.376731	13
48	.588289	5.01	.964666	.89	.623623	5.89	.376377	12
49	.588590	5.00	.964613	.89	.923976	5.89	.376024	11
50	.588890	5.00	.964560	89	.624330	5.88	.375670	10
51	9 589190	4.99	9.964507	89	9.624683	5.88	10.375317	9
52	.589489	4.99	.964454	89	.625036	5.88	.374964	8
53	.589789	4.99	.964400	.89	.625388	5.87	.374612	7
54	.590088	4.98	.964347	89	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.89	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.89	.626445	5.86	.373555	4
57	.590984	4.97	.964187	89	.626797	5 86	.373203	3
58	.591282	4.97	.964133	89	.627149	5.86	.372851	2
59	.591580	4.96	.964080	.89	627501	5.85	.372499	1
60	.591878	4.96	.964026	.89	.627852	5.85	372148	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.591878	4.96	9.964026	.89	9.627852	5.85	10.372148	60
1	.592176	4.95	.963972	.89	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.89	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.84	.371095	57
4	.593067	4.94	.963811	.90	.629255	5.84	870745	56
5	.593363	4.94	.963757	.90	.629606	5.83	.870394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	90	.630656	5.83	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.82	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.91	9.963434	.90	9.631704	5.82	10.368296	49
12	.595432	4.91	.963379	.90	.632053	5.81	.367947	48
13	.595727	4.91	.963325	.90	.632401	5.81	.367599	47
14	.596021	4.90	.963271	.90	.632750	5.81	.367250	46
15	.596315	4.90	.963217	.90	.633098	5.80	.366902	45
16	.596609	4.89	.963163	.90	.633447	5.80	.366553	44
17	.596903	4.89	.963108	.91	.633795	5.80	.366205	43
18	.597196	4.89	.963054	.91	.634143	5.79	.365857	42
19	.597490	4.88	.962999	.91	.634490	5.79	.365510	41
20	.597783	4.88	.962945	.91	.634838	5.79	.365162	40
21	9.598075	4.87	9.962890	.91	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.91	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.91	.635879	5.78	.364121	37
24	.598952	4.86	.962727	.91	.636226	5.77	.363774	36
25	.599244	4.86	.962672	.91	.636572	5.77	.363428	35
26	.599536	4.85	.962617	.91	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.91	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.91	.637611	5.76	.362389	32
29	.600409	4.84	.962453	.91	.637956	5.76	.362044	31
30	.600700	4.84	.962398	.92	.638302	5.76	.361698	30
31	9.600990	4.84	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.82	.962178	.92	.639682	5.74	.360318	26
35	.602150	4.82	.962123	.92	.640027	5.74	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.74	.359629	24
37	.602728	4.81	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.81	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.81	.961902	.92	.641404	5.73	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.72	.358253	20
41	9.603882	4.80	9.961791	.92	9.642091	5.72	10.357909	19
42	.604170	4.79	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.79	.961680	.92	.642777	5.72	.357223	17
44	.604745	4.79	.961624	.93	.643120	5.71	.356880	16
45	.605032	4 78	.961569	.93	.643463	5.71	.356537	15
46	.605319	4.78	.961513	.93	.643806	5.71	856194	14
47	.605606	4.78	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.77	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.76	.961290	.93	.645174	5.69	.354826	10
51	9.606751	4.76	9.961235	.93	9.645516	5.69	10.354484	9
52	607036	4.76	.961179	.93	.645857	5.69	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.69	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.74	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.74	.960955	.93	.647222	5.68	352778	4
57	.608461	4.74	.960899	.93	.647562	5.67	.352438	3
58	.608745	4.73	.960843	.94	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.94	.648243	5.67	.351757	1
60	.609313	4.73	.960730	.94	.648583	5.66	.351417	0
	Cosine	D.	Sine.	D.	Cotang.	D.	Tang.	M

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M	Sine.	D	Cosine.	D:	Tang.	D.	Cotang.	
0	9.609313	4.73	9.960730	.94	9.648583	5.66	10.351417	60
1	.609597	4.73	.960674	.94	.648923	5.66	.351077	59
2	.609880	4.72	.960618	.94	.649263	5.66	.350737	58
3	.610164	4.72	.960561	.94	.649602	5.66	.350398	57
4	.610447	4.71	.960505	.94	.649942	5.65	.350058	56
5	.610729	4.71	.960448	.94	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.94	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.94	.650959	5.64	.349041	53
8	.611576	4.70	.960279	.94	.651297	5.64	.348703	52
9	.611858	4.69	.960222	.94	.651636	5.64	.348364	51
10	.612140	4.69	.960165	.94	.651974	5.63	.348026	50
11	9 612421	4.69	9.960109	.95	9 652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.67	.959938	.95	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.66	.959768	.95	.654337	5.61	.345663	43
18	.614385	4.66	.959711	.95	.654674	5.61	.345326	42
19	.614665	4.66	.959654	.95	.655011	5.61	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.61	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.64	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.64	.959368	.95	.656692	5.59	.343308	36
25	.616338	4 64	.959310	.96	.657028	5.59	.342972	35
26	.616616	4.63	.959253	.96	.657364	5.59	.342636	34
27	.616894	4.63	.959195	.96	.657699	5.59	.342301	33
28	.617172	4.62	.959138	.96	.658034	5.58	.341966	32
29	.617450	4.62	.959081	.96	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.96	.658704	5.58	.341296	30
31	9.618004	4.61	9.958965	.96	9.659039	5.58	10.340961	29
32	.618281	4.61	.958908	.96	.659373	5.57	.340627	28
33	.618558	4.61	.958850	.96	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.96	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.96	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.96	.660710	5.56	.339290	24
37	.619662	4.59	.958619	.96	.661043	5.56	.338957	23
38	.619938	4.59	.958561	.96	.661377	5.56	.338623	22
39	.620213	4.59	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.57	.958329	.97	.662709	5.54	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.54	.336958	17
44	.621587	4.57	.958213	.97	.663375	5.54	.336625	16
45	.621861	4.56	.958154	.97	.663707	5.54	.336293	15
46	.622135	4.56	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.56	.958038	.97	.664371	5.53	.335629	13
48	.622682	4.55	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.965035	5.53	.334965	11
50	.623229	4.55	.957863	.97	.665366	5.52	.334634	10
51	9.623502	4.54	9.957804	.97	9.665697	5.52	10.334303	9
52	.623774	4.54	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.54	.957687	.98	.666360	5.51	.333640	7
54	.624319	4.53	.957628	.98	.666691	5.51	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.51	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.51	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.50	.332318	3
58	.625406	4.52	.957393	.98	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	.625948	4.51	.957276	.98	.668672	5.50	.331328	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.625948	4.51	9.957276	.98	9.668673	5.50	10.331327	60
1	.626219	4.51	.957217	.98	.669002	5.49	.330998	59
2	.626490	4.51	.957158	.98	.669332	5.49	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.49	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	.98	.670320	5.48	.329680	55
6	.627570	4.49	.956921	.99	.670649	5.48	.329351	54
7	.627840	4.49	.956862	.99	.670977	5.48	.329023	53
8	.628109	4.49	.956803	.99	.671306	5.47	.328694	52
9	.628378	4.48	.956744	.99	.671634	5.47	.328366	51
10	.628647	4.48	.956684	.99	.671963	5.47	.328037	50
11	9.628916	4.47	9.956625	.99	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	.99	.672619	5.46	.327381	48
13	.629453	4.47	.956506	.99	.672947	5.46	.327053	47
14	.629721	4.46	.956447	.99	.673274	5.46	.326726	46
15	.629989	4.46	.956387	.99	.673602	5.46	.326398	45
16	.630257	4.46	.956327	.99	.673929	5.45	.326071	44
17	.630524	4.46	.956268	.99	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	1.00	.674910	5.44	.325090	41
20	.631326	4.45	.956089	1.00	.675237	5.44	.324763	40
21	9.631593	4.44	9.956029	1.00	9.675564	5.44	10.324436	39
22	.631859	4.44	.955969	1.00	.675890	5.44	.324110	38
23	.632125	4.44	.955909	1.00	.676216	5.43	.323784	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.43	.955789	1.00	.676869	5.43	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.42	.322480	33
28	.633454	4.42	.955609	1.00	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.41	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.41	9.955428	1.01	9.678821	5.41	10.321179	29
32	.634514	4.40	.955368	1.01	.679146	5.41	.320854	28
33	.634778	4.40	.955307	1.01	.679471	5.41	.320529	27
34	.635042	4.40	.955247	1.01	.679795	5.41	.320205	26
35	.635306	4.39	.955186	1.01	.680120	5.40	.319880	25
36	.635570	4.39	.955126	1.01	.680444	5.40	.319556	24
37	.635834	4.39	.955065	1.01	.680768	5.40	.319232	23
38	.636097	4.38	.955005	1.01	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.01	.681416	5.39	.318584	21
40	.636623	4.38	.954883	1.01	.681740	5.39	.318260	20
41	9.636886	4.37	9.954823	1.01	9.682063	5.39	10.317937	19
42	.637148	4.37	.954762	1.01	.682387	5.39	.317613	18
43	.637411	4.37	.954701	1.01	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.01	.683033	5.38	.316967	16
45	.637935	4.36	.954579	1.01	.683356	5.38	.316644	15
46	.638197	4.36	.954518	1.02	.683679	5.38	.316321	14
47	.638458	4.36	.954457	1.02	.684001	5.37	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
51	9.639503	4.34	9.954213	1.02	9.685290	5.36	10.314710	9
52	.639764	4.34	.954152	1.02	.685612	5.36	.314388	8
53	.640024	4.34	.954090	1.02	.685934	5.36	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.36	.313745	6
55	.640544	4.33	.953968	1.02	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.32	.953845	1.02	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641584	4.32	.953722	1.03	.687861	5.34	.312139	1
60	.641842	4.31	.953660	1.03	.688182	5.34	.311818	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.641842	4.31	9.953660	1.03	9.688182	5.34	10.311818	60
1	.642101	4.31	.953599	1.03	.688502	5.34	.311498	59
2	.642360	4.31	.953537	1.03	.688823	5.34	.311177	58
3	.642618	4.30	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.03	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.30	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.29	.953228	1.03	.690423	5.33	.309577	53
8	.643908	4.29	.953166	1.03	.690742	5.32	.309258	52
9	.644165	4.29	.953104	1.03	.691062	5.32	.308938	51
10	.644423	4.28	.953042	1.03	.691381	5.32	.308619	50
11	9.644680	4.28	9.952980	1.04	9.691700	5.31	10.308300	49
12	.644936	4.28	.952918	1.04	.692019	5.31	.307981	48
13	.645193	4.27	.952855	1.04	.692338	5.31	.307662	47
14	.645450	4.27	.952793	1.04	.692656	5.31	.307344	46
15	.645706	4.27	.952731	1.04	.692975	5.31	.307025	45
16	.645962	4.26	.952669	1.04	.693293	5.30	.306707	44
17	.646218	4.26	.952606	1.04	.693612	5.30	.306388	43
18	.646474	4.26	.952544	1.04	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.04	.694248	5.30	.305752	41
20	.646984	4.25	.952419	1.04	.694566	5.29	.305434	40
21	9.647240	4.25	9.952356	1.04	9.694883	5.29	10.305117	39
22	.647494	4.24	.952294	1.04	.695201	5.29	.304799	38
23	.647749	4.24	.952231	1.04	.695518	5.29	.304482	37
24	.648004	4.24	.952168	1.05	.695836	5.29	.304164	36
25	.648258	4.24	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.28	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	.649527	4.22	.951791	1.05	.697736	5.27	.302264	30
31	9.649781	4.22	9.951728	1.05	9.698053	5.27	10.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.21	.951602	1.05	.698685	5.26	.301315	27
34	.650539	4.21	.951539	1.05	.699001	5.26	.300999	26
35	.650792	4.21	.951476	1.05	.699316	5.26	.300684	25
36	.651044	4.20	.951412	1.05	.699632	5.26	.300368	24
37	.651297	4.20	.951349	1.06	.699947	5.26	.300053	23
38	.651549	4.20	.951286	1.06	.700263	5.25	.299737	22
39	.651800	4.19	.951222	1.06	.700578	5.25	.299422	21
40	.652052	4.19	.951159	1.06	.700893	5.25	.299107	20
41	9.652304	4.19	9.951096	1.06	9.701208	5.24	10.298792	19
42	.652555	4.18	.951032	1.06	.701523	5.24	.298477	18
43	.652806	4.18	.950968	1.06	.701837	5.24	.298163	17
44	.653057	4.18	.950905	1.06	.702152	5.24	.297848	16
45	.653308	4.18	.950841	1.06	.702466	5.24	.297534	15
46	.653558	4.17	.950778	1.06	.702780	5.23	.297220	14
47	.653808	4.17	.950714	1.06	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.06	.703409	5.23	.296591	12
49	.654309	4.16	.950586	1.06	.703723	5.23	.296277	11
50	.654558	4.16	.950522	1.07	.704036	5.22	.295964	10
51	9.654808	4.16	9.950458	1.07	9.704350	5.22	10.295650	9
52	.655058	4.16	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704977	5.22	.295023	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.21	.294397	5
56	.656054	4.14	.950138	1.07	.705916	5.21	.294084	4
57	.656302	4.14	.950074	1.07	.706228	5.21	.293772	3
58	.656551	4.14	.950010	1.07	.706541	5.21	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.21	.293146	1
60	.657047	4.13	.949881	1.07	.707166	5.20	.292834	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	10.292834	60
1	.657295	4.13	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.12	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.19	.291586	56
5	.658284	4.12	.949558	1.08	.708726	5.19	.291274	55
6	.658531	4.11	.949494	1.08	.709037	5.19	.290963	54
7	.658778	4.11	.949429	1.08	.709349	5.19	.290651	53
8	.659025	4.11	.949364	1.08	.709660	5.19	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.09	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.09	.949040	1.08	.711215	5.18	.288785	47
14	.660501	4.09	.948975	1.08	.711525	5.17	.288475	46
15	.660746	4.09	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.09	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.09	.712766	5.16	.287234	42
19	.661726	4.07	.948650	1.09	.713076	5.16	.286924	41
20	.661970	4.07	.948584	1.09	.713386	5.16	.286614	40
21	9.662214	4.07	9.948519	1.09	9.713696	5.16	10.286304	39
22	.662459	4.07	.948454	1.09	.714005	5.16	.285995	38
23	.662703	4.06	.948388	1.09	.714314	5.15	.285686	37
24	.662946	4.06	.948323	1.09	.714624	5.15	.285376	36
25	.663190	4.06	.948257	1.09	.714933	5.15	.285067	35
26	.663433	4.05	.948192	1.09	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.09	.715551	5.14	.284449	33
28	.663920	4.05	.948060	1.09	.715860	5.14	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.14	.283832	31
30	.664406	4.04	.947929	1.10	.716477	5.14	.283523	30
31	9.664648	4.04	9.947863	1.10	9.716785	5.14	10.283215	29
32	.664891	4.04	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.10	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.10	.718017	5.13	.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281670	24
37	.666100	4.02	.947467	1.10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.12	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	.666824	4.01	.947269	1.10	.719555	5.12	.280445	20
41	9.667065	4.01	9.947203	1.10	9.719862	5.12	10.280138	19
42	.667305	4.01	.947136	1.11	.720169	5.11	.279831	18
43	.667546	4.01	.947070	1.11	.720476	5.11	.279524	17
44	.667786	4.00	.947004	1.11	.720783	5.11	.279217	16
45	.668027	4.00	.946937	1.11	.721089	5.11	.278911	15
46	.668267	4.00	.946871	1.11	.721396	5.11	.278604	14
47	.668506	3.99	.946804	1.11	.721702	5.10	.278298	13
48	.668746	3.99	.946738	1.11	.722009	5.10	.277991	12
49	.668986	3.99	.946671	1.11	.722315	5.10	.277685	11
50	.669225	3.99	.946604	1.11	.722621	5.10	.277379	10
51	9.669464	3.98	9.946538	1.11	9.722927	5.10	10.277073	9
52	.669703	3.98	.946471	1.11	.723232	5.09	.276768	8
53	.669942	3.98	.946404	1.11	.723538	5.09	.276462	7
54	.670181	3.97	.946337	1.11	.723844	5.09	.276156	6
55	.670419	3.97	.946270	1.12	.724149	5.09	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.09	.275546	4
57	.670896	3.97	.946136	1.12	.724759	5.08	.275241	3
58	.671134	3.96	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.96	.946002	1.12	.725369	5.08	.274631	1
60	.671609	3.96	.945935	1.12	.725674	5.08	.274326	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	10.274326	60
1	.671847	3.95	.945868	1.12	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.12	.726892	5.07	.273108	56
5	.672795	3.94	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.94	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.94	.945464	1.13	.727805	5.06	.272195	53
8	.673505	3.94	.945396	1.13	.728109	5.06	.271891	52
9	.673741	3.93	.945328	1.13	.728412	5.06	.271588	51
10	.673977	3.93	.945261	1.13	.728716	5.06	.271284	50
11	9.674213	3.93	9.945193	1.13	9.729020	5.06	10.270980	49
12	.674448	3.92	.945125	1.13	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.92	.944990	1.13	.729929	5.05	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.05	.269767	45
16	.675390	3.91	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.91	.944786	1.13	.730838	5.04	.269162	43
18	.675859	3.91	.944718	1.13	.731141	5.04	.268859	42
19	.676094	3.91	.944650	1.13	.731444	5.04	.268556	41
20	.676328	3.90	.944582	1.14	.731746	5.04	.268254	40
21	9.676562	3.90	9.944514	1.14	9.732048	5.04	10.267952	39
22	.676796	3.90	.944446	1.14	.732351	5.03	.267649	38
23	.677030	3.90	.944377	1.14	.732653	5.03	.267347	37
24	.677264	3.89	.944309	1.14	.732955	5.03	.267045	36
25	.677498	3.89	.944241	1.14	.733257	5.03	.266743	35
26	.677731	3.89	.944172	1.14	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.14	.733860	5.02	.266140	33
28	.678197	3.88	.944036	1.14	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.14	.734463	5.02	.265537	31
30	.678663	3.88	.943899	1.14	.734764	5.02	.265236	30
31	9.678895	3.87	9.943830	1.14	9.735066	5.02	10.264934	29
32	.679128	3.87	.943761	1.14	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.01	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.01	.264031	26
35	.679824	3.86	.943555	1.15	.736269	5.01	.263731	25
36	.680056	3.86	.943486	1.15	.736570	5.01	.263430	24
37	.680288	3.86	.943417	1.15	.736871	5.01	.263129	23
38	.680519	3.85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.85	.943279	1.15	.737471	5.00	.262529	21
40	.680982	3.85	.943210	1.15	.737771	5.00	.262229	20
41	9.681213	3.85	9.943141	1.15	9.738071	5.00	10.261929	19
42	.681443	3.84	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.84	.943003	1.15	.738671	4.99	.261329	17
44	.681905	3.84	.942934	1.15	.738971	4.99	.261029	16
45	.682135	3.84	.942864	1.15	.739271	4.99	.260729	15
46	.682365	3.83	.942795	1.16	.739570	4.99	.260430	14
47	.682595	3.83	.942726	1.16	.739870	4.99	.260130	13
48	.682825	3.83	.942656	1.16	.740169	4.99	.259831	12
49	.683055	3.83	.942587	1.16	.740468	4.98	.259532	11
50	.683284	3.82	.942517	1.16	.740767	4.98	.259233	10
51	9.683514	3.82	9.942448	1.16	9.741066	4.98	10.258934	9
52	.683743	3.82	.942378	1.16	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.16	.741664	4.98	.258336	7
54	.684201	3.81	.942239	1.16	.741962	4.97	.258038	6
55	.684430	3.81	.942169	1.16	.742261	4.97	.257739	5
56	.684658	3.81	.942099	1.16	.742559	4.97	.257441	4
57	.684887	3.80	.942029	1.16	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.16	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	.685571	3.80	.941819	1.17	.743752	4.96	.256248	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	10.256248	60
1	.685799	3.79	.941749	1.17	.744050	4.96	.255950	59
2	.686027	3.79	.941679	1.17	.744348	4.96	.255652	58
3	.686254	3.79	.941609	1.17	.744645	4.96	.255355	57
4	.686482	3.79	.941539	1.17	.744943	4.96	.255057	56
5	.686709	3.78	.941469	1.17	.745240	4.96	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.78	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.17	.746132	4.95	.253868	52
9	.687616	3.77	.941187	1.17	.746429	4.95	.253571	51
10	.687843	3.77	.941117	1.17	.746726	4.95	.253274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.94	10.252977	49
12	.688295	3.77	.940975	1.18	.747319	4.94	.252681	48
13	.688521	3.76	.940905	1.18	.747616	4.94	.252384	47
14	.688747	3 76	.940834	1.18	.747913	4.94	.252087	46
15	.688972	3.76	.940763	1.18	.748209	4.94	.251791	45
16	.689198	3.76	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311	40
21	9.690323	3.74	9.940338	1.18	9.749985	4.93	10.250015	39
22	.690548	3.74	.940267	1 18	.750281	4.92	.249719	38
23	.690772	3.74	.940196	1.18	.750576	4.92	.249424	37
24	.690996	3.74	.940125	1.19	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.19	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.19	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.19	.751757	4.92	.248243	33
28	.691892	3.73	.939840	1.19	.752052	4.91	.247948	32
29	.692115	3.72	.939768	1.19	.752347	4.91	.247653	31
30	.692339	3.72	.939697	1 19	.752642	4.91	.247358	30
31	9.692562	3.72	9.939625	1.19	9.752937	4.91	10.247063	29
32	.692785	3.71	.939554	1.19	.753231	4.91	.246769	28
33	.693008	3.71	.939482	1.19	.753526	4.91	.246474	27
34	.693231	3.71	.939410	1.19	.753820	4.90	.246180	26
35	.693453	3.71	.939339	1.19	.754115	4.90	.245885	25
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.20	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	.694564	3.69	.938980	1.20	.755585	4.89	.244415	20
41	9.694786	3.69	9.938908	1.20	9.755878	4.89	10.244122	19
42	.695007	3.69	.938836	1.20	.756172	4.89	.243828	18
43	.695229	3.69	.938763	1.20	.756465	4.89	.243535	17
44	.695450	3.68	.938691	1.20	.756759	4.89	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.89	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.20	.757638	4.88	.242362	13
48	.696334	3.67	.938402	1.21	.757931	4.88	.242069	12
49	.696554	3.67	.938330	1.21	.758224	4.88	.241776	11
50	.696775	3.67	.938258	1.21	.758517	4.88	.241483	10
51	9.696995	3.67	9.938185	1.21	9.758810	4.88	10.241190	9
52	.697215	3.66	.938113	1.21	.759102	4.87	.240898	8
53	.697435	3.66	.938040	1.21	.759395	4.87	.240605	7
54	.697654	3.66	.937967	1.21	.759687	4.87	.240313	6
55	.697874	3.66	.937895	1.21	.759979	4.87	.240021	5
56	.698094	3.65	.937822	1.21	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.21	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.21	.760856	4.86	.239144	2
59	.698751	3.65	.937604	1.21	.761148	4.86	.238852	1
60	.698970	3.64	.937531	1.21	.761439	4.86	.238561	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.698970	3.64	9.937531	1.21	9.761439	4.86	10.238561	60
1	.699189	3.64	.937458	1.22	.761731	4.86	.238269	59
2	.699407	3.64	.937385	1.22	.762023	4.86	.237977	58
3	.699626	3.64	.937312	1.22	.762314	4.86	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.63	.936946	1.22	.763770	4.85	.236230	52
9	.700933	3.62	.936872	1.22	.764061	4.85	.235939	51
10	.701151	3.62	.936799	1.22	.764352	4.84	.235648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.84	10.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.84	.235067	48
13	.701802	3.61	.936578	1.23	.765224	4.84	.234776	47
14	.702019	3.61	.936505	1.23	.765514	4.84	.234486	46
15	.702236	3.61	.936431	1.23	.765805	4.84	.234195	45
16	.702452	3.61	.936357	1.23	.766095	4.84	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	.703317	3.60	.936062	1.23	.767255	4.83	.232745	40
21	9.703533	3.59	9.935988	1.23	9.767545	4.83	10.232455	39
22	.703749	3.59	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.59	.935840	1.23	.768124	4.82	.231876	37
24	.704179	3.59	.935766	1.24	.768413	4.82	.231587	36
25	.704395	3.59	.935692	1.24	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.24	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.24	.769281	4.82	.230719	33
28	.705040	3.58	.935469	1.24	.769570	4.82	.230430	32
29	.705254	3.58	.935395	1.24	.769860	4.81	.230140	31
30	.705469	3.57	.935320	1.24	.770148	4.81	.229852	30
31	9.705683	3.57	9.935246	1.24	9.770437	4.81	10.229563	29
32	.705898	3.57	.935171	1.24	.770726	4.81	.229274	28
33	.706112	3.57	.935097	1.24	.771015	4.81	.228985	27
34	.706326	3.56	.935022	1.24	.771303	4.81	.228697	26
35	.706539	3.56	.934948	1.24	.771592	4.81	.228408	25
36	.706753	3.56	.934873	1.24	.771880	4.80	.228120	24
37	.706967	3.56	.934798	1.25	.772168	4.80	.227832	23
38	.707180	3.55	.934723	1.25	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	.707606	3.55	.934574	1.25	.773033	4.80	.226967	20
41	9.707819	3.55	9.934499	1.25	9.773321	4.80	10.226679	19
42	.708032	3.54	.934424	1.25	.773608	4.79	.226392	18
43	.708245	3.54	.934349	1.25	.773896	4.79	.226104	17
44	.708458	3.54	.934274	1.25	.774184	4.79	.225816	16
45	.708670	3.54	.934199	1.25	.774471	4.79	.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.79	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.79	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.79	.224667	12
49	.709518	3.53	.933898	1.26	.775621	4.78	.224379	11
50	.709730	3.53	.933822	1.26	.775908	4.78	.224092	10
51	9.709941	3.52	9.933747	1.26	9.776195	4.78	10.223805	9
52	.710153	3.52	.933671	1.26	.776482	4.78	.223518	8
53	.710364	3.52	.933596	1.26	.776769	4.78	.223231	7
54	.710575	3.52	.933520	1.26	.777055	4.78	.222945	6
55	.710786	3.51	.933445	1.26	.777342	4.78	.222658	5
56	.710997	3.51	.933369	1.26	.777628	4.77	.222372	4
57	.711208	3.51	.933293	1.26	.777915	4.77	.222085	3
58	.711419	3.51	.933217	1.26	.778201	4.77	.221799	2
59	.711629	3.50	.933141	1.26	.778487	4.77	.221512	1
60	.711839	3.50	.933066	1.26	.778774	4.77	.221226	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.711839	3.50	9.933066	1.26	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.50	.932914	1.27	.779346	4.76	.220654	58
3	.712469	3.49	.932838	1.27	.779632	4.76	.220368	57
4	.712679	3.49	.932762	1.27	.779918	4.76	.220082	56
5	.712889	3.49	.932685	1.27	.780203	4.76	.219797	55
6	.713098	3.49	.932609	1.27	.780489	4.76	.219511	54
7	.713308	3.49	.932533	1.27	.780775	4.76	.219225	53
8	.713517	3.48	.932457	1.27	.781060	4.76	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.48	9.932228	1.27	9.781916	4.75	10.218084	49
12	.714352	3.47	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.47	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.28	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.46	.931768	1.28	.783626	4.74	.216374	43
18	.715602	3.46	.931691	1.28	.783910	4.74	.216090	42
19	.715809	3.46	.931614	1.28	.784195	4.74	.215805	41
20	.716017	3.46	.931537	1.28	.784479	4.74	.215521	40
21	9.716224	3.45	9.931460	1.28	9.784764	4.74	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.74	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.29	.785616	4.73	.214384	36
25	.717053	3.45	.931152	1.29	.785900	4.73	.214100	35
26	.717259	3.44	.931075	1.29	.786184	4.73	.213816	34
27	.717466	3.44	.930998	1.29	.786468	4.73	.213532	33
28	.717673	3.44	.930921	1.29	.786752	4.73	.213248	32
29	.717879	3.44	.930843	1.29	.787036	4.73	.212964	31
30	.718085	3.43	.930766	1.29	.787319	4.72	.212681	30
31	9.718291	3.43	9.930688	1.29	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.29	.787886	4.72	.212114	28
33	.718703	3.43	.930533	1.29	.788170	4.72	.211830	27
34	.718909	3.43	.930456	1.29	.788453	4.72	.211547	26
35	.719114	3.42	.930378	1.29	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.30	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.71	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.71	.210415	22
39	.719935	3.41	.930067	1.30	.789868	4.71	.210132	21
40	.720140	3.41	.929989	1.30	.790151	4.71	.209849	20
41	9.720345	3.41	9.929911	1.30	9.790433	4.71	10.209567	19
42	.720549	3.41	.929833	1.30	.790716	4.71	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.71	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.71	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.70	.208437	15
46	.721366	3.40	.929521	1.30	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.39	.929364	1.31	.792410	4.70	.207590	12
49	.721978	3.39	.929286	1.31	.792692	4.70	.207308	11
50	.722181	3.39	.929207	1.31	.792974	4.70	.207026	10
51	9.722385	3.39	9.929129	1.31	9.793256	4.70	10.206744	9
52	.722588	3.39	.929050	1.31	.793538	4.69	.206462	8
53	.722791	3.38	.928972	1.31	.793819	4.69	.206181	7
54	.722994	3.38	.928893	1.31	.794101	4.69	.205899	6
55	.723197	3.38	.928815	1.31	.794383	4.69	.205617	5
56	.723400	3.38	.928736	1.31	.794664	4.69	.205336	4
57	.723603	3.37	.928657	1.31	.794945	4.69	.205055	3
58	.723805	3.37	.928578	1.31	.795227	4.69	.204773	2
59	.724007	3.37	.928499	1.31	.795508	4.68	.204492	1
60	.724210	3.37	.928420	1.31	.795789	4.68	.204211	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

M	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	10.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.36	.928263	1.32	.796351	4.68	.203649	58
3	.724816	3.36	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.36	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.36	.928025	1.32	.797194	4.68	.202806	55
6	.725420	3.35	.927946	1.32	.797475	4.68	.202525	54
7	.725622	3.35	.927867	1.32	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.32	.798596	4.67	.201404	50
11	9.726426	3.34	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.34	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.34	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.34	.927310	1.33	.799717	4.67	.200283	46
15	.727228	3.34	.927231	1.33	.799997	4.66	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.66	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.66	.199443	43
18	.727828	3.33	.926991	1.33	.800836	4.66	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.66	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.66	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.66	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.66	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.34	.802513	4.65	.197487	36
25	.729223	3.31	.926431	1.34	.802792	4.65	.197208	35
26	.729422	3.31	.926351	1.34	.803072	4.65	.196928	34
27	.729621	3.31	.926270	1.34	.803351	4.65	.196649	33
28	.729820	3.31	.926190	1.34	.803630	4.65	.196370	32
29	.730018	3.30	.926110	1.34	.803908	4.65	.196092	31
30	.730216	3.30	.926029	1.34	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.34	9.804466	4.64	10.195534	29
32	.730613	3.30	.925868	1.34	.804745	4.64	.195255	28
33	.730811	3.30	.925788	1.34	.805023	4.64	.194997	27
34	.731009	3.29	.925707	1.34	.805302	4.64	.194698	26
35	.731206	3.29	.925626	1.34	.805580	4.64	.194420	25
36	.731404	3.29	.925545	1.35	.805859	4.64	.194141	24
37	.731602	3.29	.925465	1.35	.806137	4.64	.193863	23
38	.731799	3.29	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.28	.924979	1.35	.807805	4.63	.192195	17
44	.732980	3.27	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.63	.191639	15
46	.733373	3.27	.924735	1.36	.808638	4.62	.191362	14
47	.733569	3.27	.924654	1.36	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.36	.809193	4.62	.190807	12
49	.733961	3.26	.924491	1.36	.809471	4.62	.190529	11
50	.734157	3.26	.924409	1.36	809748	4.62	.190252	10
51	9.734353	3.26	9.924328	1.36	9.810025	4.62	10.189975	9
52	.734549	3.26	.924246	1.36	.810302	4.62	.189698	8
53	.734744	3.25	.924164	1.36	.810580	4.62	.189420	7
54	.734939	3.25	.924083	1.36	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.36	.811134	4.61	.188866	5
56	.735330	3.25	.923919	1.36	.811410	4.61	.188590	4
57	.735525	3.25	.923837	1.36	.811687	4.61	.188313	3
58	.735719	3.24	.923755	1.37	.811964	4.61	.188036	2
59	.735914	3.24	.923673	1.37	.812241	4.61	.187759	1
60	.736109	3.24	.923591	1.37	.812517	4.61	.187483	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	10.187482	60
1	.736303	3.24	.923509	1.37	.812794	4.61	.187206	59
2	.736498	3.24	.923427	1.37	.813070	4.61	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.37	.813899	4.60	.186101	55
6	.737274	3.23	.923098	1.37	.814175	4.60	.185825	54
7	.737467	3.23	.923016	1.37	.814452	4.60	.185548	53
8	.737661	3.22	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.37	.815004	4.60	.184996	51
10	.738048	3.22	.922768	1.38	.815279	4.60	.184721	50
11	9.738241	3.22	9.922686	1.38	9.815555	4.59	10.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.59	.184169	48
13	.738627	3.21	.922520	1.38	.816107	4.59	.183893	47
14	.738820	3.21	.922438	1.38	.816382	4.59	.183618	46
15	.739013	3.21	.922355	1.38	.816658	4.59	.183342	45
16	.739206	3.21	.922272	1.38	.816933	4.59	.183067	44
17	.739398	3.21	.922189	1.38	.817209	4.59	.182791	43
18	.739590	3.20	.922106	1.38	.817484	4.59	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.59	.182241	41
20	.739975	3.20	.921940	1.38	.818035	4.58	.181965	40
21	9.740167	3.20	9.921857	1.39	9.818310	4.58	10.181690	39
22	.740359	3.20	.921774	1 39	.818585	4.58	.181415	38
23	.740550	3.19	.921691	1.39	.818860	4.58	.181140	37
24	.740742	3.19	.921607	1.39	.819135	4.58	.180865	36
25	.740934	3.19	.921524	1.39	.819410	4.58	.180590	35
26	.741125	3.19	.921441	1.39	.819684	4.58	.180316	34
27	.741316	3.19	.921357	1.39	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.39	.820234	4.58	.179766	32
29	.741699	3.18	.921190	1.39	.820508	4.57	.179492	31
30	.741889	3.18	.921107	1·39	.820783	4.57	.179217	30
31	9.742080	3.18	9.921023	1.39	9.821057	4.57	10.178943	29
32	.742271	3.18	.920939	1.40	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.17	.920688	1.40	.822154	4.57	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.16	.920436	1.40	.822977	4.56	.177023	22
39	.743602	3.16	.920352	1.40	.823250	4.56	.176750	21
40	.743792	3.16	.920268	1.40	.823524	4.56	.176476	20
41	9.743982	3.16	9.920184	1.40	9.823798	4.56	10.176202	19
42	.744171	3.16	.920099	1.40	.824072	4.56	.175928	18
43	.744361	3.15	.920015	1.40	.824345	4.56	.175655	17
44	.744550	3.15	.919931	1.41	.824619	4.56	.175381	16
45	.744739	3.15	.919846	1.41	.824893	4.56	.175107	15
46	.744928	3.15	.919762	1.41	.825166	4.56	.174834	14
47	.745117	3.15	.919677	1 41	.825439	4.55	.174561	13
48	.745306	3.14	.919593	1 41	.825713	4.55	.174287	12
49	.745494	3.14	.919508	1.41	.825986	4.55	.174014	11
50	.745683	3.14	.919424	1.41	.826259	4.55	.173741	10
51	9.745871	3.14	9.919339	1.41	9.826532	4.55	10.173468	9
52	.746059	3.14	.919254	1 41	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.41	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.41	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.41	.827624	4.55	.172376	5
56	.746812	3 13	.918915	1.42	.827897	4.54	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.54	.171830	3
58	.747187	3.12	.918745	1.42	.828442	4.54	.171558	2
59	.747374	3.12	.918659	1 42	.828715	4.54	.171285	1
60	.747562	3.12	.918574	1.42	.828987	4.54	.171013	0
	Cosine	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	10.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.54	.170740	59
2	.747936	3.12	.918404	1.42	.829532	4.54	.170468	58
3	.748123	3.11	.918318	1.42	.829805	4.54	.170195	57
4	.748310	3.11	.918233	1.42	.830077	4.54	.169923	56
5	.748497	3.11	.918147	1.42	.830349	4.53	.169651	55
6	.748683	3.11	.918062	1.42	.830621	4.53	.169379	54
7	.748870	3.11	.917976	1.43	.830893	4.53	.169107	53
8	.749056	3.10	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	.749429	3.10	.917719	1.43	.831709	4.53	.168291	50
11	9.749615	3.10	9.917634	1.43	9.831981	4.53	10.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.09	.917462	1.43	.832525	4.53	.167475	47
14	.750172	3.09	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.09	.917290	1.43	.833068	4.52	.166932	45
16	.750543	3.09	.917204	1.43	.833339	4.52	.166661	44
17	.750729	3.09	.917118	1.44	.833611	4.52	.166389	43
18	.750914	3.08	.917032	1.44	.833882	4.52	.166118	42
19	.751099	3.08	.916946	1.44	.834154	4.52	.165846	41
20	.751284	3.08	.916859	1.44	.834425	4.52	.165575	40
21	9.751469	3.08	9.916773	1.44	9.834696	4.52	10.165304	39
22	.751654	3.08	.916687	1.44	.834967	4.52	.165033	38
23	.751839	3.08	.916600	1.44	.835238	4.52	.164762	37
24	.752023	3.07	.916514	1.44	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.44	.835780	4.51	.164220	35
26	.752392	3.07	.916341	1.44	.836051	4.51	.163949	34
27	.752576	3.07	.916254	1.44	.836322	4.51	.163678	33
28	.752760	3.07	.916167	1.45	.836593	4.51	.163407	32
29	.752944	3.06	.916081	1.45	.836864	4.51	.163136	31
30	.753128	3.06	.915994	1.45	.837134	4.51	.162866	30
31	9.753312	3.06	9.915907	1.45	9.837405	4.51	10.162595	29
32	.753495	3.06	.915820	1.45	.837675	4.51	.162325	28
33	.753679	3.06	.915733	1.45	.837946	4.51	.162054	27
34	.753862	3.05	.915646	1.45	.838216	4.51	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.50	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.45	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.50	.160703	22
39	.754778	3.04	.915210	1.45	.839568	4.50	.160432	21
40	.754960	3.04	.915123	1.46	.839838	4.50	.160162	20
41	9.755143	3.04	9.915035	1.46	9.840108	4.50	10.159892	19
42	.755326	3.04	.914948	1.46	.840378	4.50	.159622	18
43	.755508	3.04	.914860	1.46	.840647	4.50	.159353	17
44	.755690	3.04	.914773	1.46	.840917	4.49	.159083	16
45	.755872	3.03	.914685	1.46	.841187	4.49	.158813	15
46	.756054	3.03	.914598	1.46	.841457	4.49	.158543	14
47	.756236	3.03	.914510	1.46	.841726	4.49	.158274	13
48	.756418	3.03	.914422	1.46	.841996	4.49	.158004	12
49	.756600	3.03	.914334	1.46	.842266	4.49	.157734	11
50	.756782	3.02	.914246	1.47	842535	4.49	.157465	10
51	9.756963	3.02	9.914158	1.47	9.842805	4.49	10.157195	9
52	.757144	3.02	.914070	1.47	.843074	4.49	.156926	8
53	.757326	3.02	.913982	1.47	.843343	4.49	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.49	.156388	6
55	.757688	3.01	.913806	1.47	.843882	4.48	.156118	5
56	.757869	3.01	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.01	.913630	1.47	.844420	4.48	.155580	3
58	.758230	3.01	.913541	1.47	.844689	4.48	.155311	2
59	.758411	3.01	.913453	1.47	.844958	4.48	.155042	1
60	.758591	3.01	.913365	1.47	.845227	4.48	.154773	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.47	.845496	4.48	.154504	59
2	.758952	3.00	.913187	1.48	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.48	.846302	4.48	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.47	.153430	55
6	.759672	2.99	.912833	1.48	.846839	4.47	.153161	54
7	.759852	2.99	.912744	1.48	.847107	4.47	.152893	53
8	.760031	2.99	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.99	.912566	1.48	.847644	4.47	.152356	51
10	.760390	2.99	.912477	1.48	.847913	4.47	.152087	50
11	9.760569	2.98	9.912388	1.48	9.848181	4.47	10.151819	49
12	.760748	2.98	.912299	1.49	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.49	.848717	4.47	.151283	47
14	.761106	2.98	.912121	1.49	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.49	.849254	4.47	.150746	45
16	.761464	2.98	.911942	1.49	.849522	4.47	.150478	44
17	.761642	2.97	.911853	1.49	.849790	4.46	.150210	43
18	.761821	2.97	.911763	1.49	.850058	4.46	.149942	42
19	.761999	2.97	.911674	1.49	.850325	4.46	.149675	41
20	.762177	2.97	.911584	1.49	.850593	4.46	.149407	40
21	9.762356	2.97	9.911495	1.49	9.850861	4.46	10.149139	39
22	.762534	2.96	.911405	1.49	.851129	4.46	.148871	38
23	.762712	2.96	.911315	1.50	.851396	4.46	.148604	37
24	.762889	2.96	.911226	1.50	.851664	4.46	.148336	36
25	.763067	2.96	.911136	1.50	.851931	4.46	.148069	35
26	.763245	2.96	.911046	1.50	.852199	4.46	.147801	34
27	.763422	2.96	.910956	1.50	.852466	4.46	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.45	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	.763954	2.95	.910686	1.50	.853268	4.45	.146732	30
31	9.764131	2.95	9.910596	1.50	9.853535	4.45	10.146465	29
32	.764308	2.95	.910506	1.50	.853802	4.45	.146198	28
33	.764485	2.94	.910415	1.50	.854069	4.45	.145931	27
34	.764662	2.94	.910325	1.51	.854336	4.45	.145664	26
35	.764838	2.94	.910235	1.51	.854603	4.45	.145397	25
36	.765015	2.94	.910144	1.51	.854870	4.45	.145130	24
37	.765191	2.94	.910054	1.51	.855137	4.45	.144863	23
38	.765367	2.94	.909963	1.51	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.51	.855671	4.44	.144329	21
40	.765720	2.93	.909782	1.51	.855938	4.44	.144062	20
41	9.765896	2.93	9.909691	1.51	9.856204	4.44	10.143796	19
42	.766072	2.93	.909601	1.51	.856471	4.44	.143529	18
43	.766247	2.93	.909510	1.51	.856737	4.44	.143263	17
44	.766423	2.93	.909419	1.51	.857004	4.44	.142996	16
45	.766598	2.92	.909328	1.52	.857270	4.44	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.44	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.44	.142197	13
48	.767124	2.92	.909055	1.52	.858069	4.44	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.44	.141664	11
50	.767475	2.91	.908873	1.52	.858602	4.43	.141398	10
51	9.767649	2.91	9.908781	1.52	9.858868	4.43	10.141132	9
52	.767824	2.91	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.91	.908599	1.52	.859400	4.43	.140600	7
54	.768173	2.91	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.90	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.90	.908324	1.53	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.43	.139270	2
59	.769045	2.90	.908049	1.53	.860995	4.43	.139005	1
60	.769219	2.90	.907958	1.53	.861261	4.43	.138739	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60
1	.769393	2.89	.907866	1.53	.861527	4.43	.138473	59
2	.769566	2.89	.907774	1.53	.861792	4.42	.138208	58
3	.769740	2.89	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.89	.907590	1.53	.862323	4.42	.137677	56
5	.770087	2.89	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.54	.863119	4.42	.136881	53
8	.770606	2.88	.907222	1.54	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.54	.863650	4.42	.136350	51
10	.770952	2.88	.907037	1.54	.863915	4.42	.136085	50
11	9.771125	2.88	9.906945	1.54	9.864180	4.42	10.135820	49
12	.771298	2.87	.906852	1.54	.864445	4.42	.135555	48
13	.771470	2.87	.906760	1.54	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.54	.864975	4.41	.135025	46
15	.771815	2.87	.906575	1.54	.865240	4.41	.134760	45
16	.771987	2.87	.906482	1.54	.865505	4.41	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.41	.134230	43
18	.772331	2.86	.906296	1 55	.866035	4.41	.133965	42
19	.772503	2.86	.906204	1.55	.866300	4.41	.133700	41
20	.772675	2.86	.906111	1.55	.866564	4.41	.133436	40
21	9.772847	2.86	9.906018	1.55	9.866829	4.41	10.133171	39
22	.773018	2.86	.905925	1.55	.867094	4.41	.132906	38
23	.773190	2.86	.905832	1.55	.867358	4.41	.132642	37
24	.773361	2.85	.905739	1.55	.867623	4.41	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.41	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2.85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.56	.868680	4.40	.131320	32
29	.774217	2.85	.905272	1.56	.868945	4.40	.131055	31
30	.774388	2.84	.905179	1.56	.869209	4.40	.130794	30
31	9.774558	2.84	9.905085	1.56	9.869473	4.40	10.130527	29
32	.774729	2.84	.904992	1 56	.869737	4.40	.130263	28
33	.774899	2.84	.904898	1.56	.870001	4.40	.129999	27
34	.775070	2.84	.904804	1.56	.870265	4.40	.129735	26
35	.775240	2.84	.904711	1.56	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.56	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.56	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	.776090	2.83	.904241	1.57	.871849	4.39	.128151	20
41	9.776259	2.83	9.904147	1.57	9.872112	4.39	10.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.39	.127624	18
43	.776598	2.82	.903959	1.57	.872640	4.39	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.39	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.39	.126833	15
46	.777106	2.82	.903676	1.57	.873430	4.39	.126570	14
47	.777275	2.81	.903581	1.57	.873694	4.39	.126306	13
48	.777444	2.81	.903487	1.57	.873957	4.39	.126043	12
49	.777613	2.81	.903392	1.58	.874220	4.39	.125780	11
50	.777781	2.81	.903298	1.58	874484	4.39	.125516	10
51	9.777950	2.81	9.903203	1.58	9.874747	4.39	10.125253	9
52	.778119	2.81	.903108	1.58	.875010	4.39	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4.38	.124727	7
54	.778455	2.80	.902919	1.58	.875536	4.38	.124464	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.80	.902539	1.59	.876589	4.38	.123411	2
59	.779295	2.79	.902444	1.59	.876851	4.38	.123149	1
60	.779463	2.79	.902349	1.59	.877114	4.38	.122886	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

37° miles

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.779463	2.79	9.902349	1.59	9.877114	4.38	10.122886	60
1	.779631	2.79	.902258	1.59	.877377	4.38	.122623	59
2	.779798	2.79	.902158	1.59	.877640	4.38	.122360	58
3	.779966	2.79	.902068	1.59	.877903	4.38	.122097	57
4	.780133	2.79	.901967	1.59	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.59	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.59	.878691	4.38	.121309	54
7	.780634	2.78	.901681	1.59	.878953	4.37	.121047	53
8	.780801	2.78	.901585	1.59	.879216	4.37	.120784	52
9	.780968	2.78	.901490	1.59	.879478	4.37	.120522	51
10	.781134	2.78	.901394	1.60	.879741	4.37	.120259	50
11	9.781301	2.77	9.901298	1.60	9.880003	4.37	10.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.37	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.37	.118686	44
17	.782298	2.76	.900722	1.60	.881576	4.37	.118424	43
18	.782464	2.76	.900626	1.60	.881839	4.37	.118161	42
19	.782630	2.76	.900529	1.60	.882101	4.37	.117899	41
20	.782796	2.76	.900433	1.61	.882363	4.36	.117637	40
21	9.782961	2.76	9.900337	1.61	9.882625	4.36	10.117375	39
22	.783127	2.76	.900240	1.61	.882887	4.36	.117113	38
23	.783292	2.75	.900144	1.61	.883148	4.36	.116852	37
24	.783458	2.75	.900047	1.61	.883410	4.36	.116590	36
25	.783623	2.75	.899951	1.61	.883672	4.36	.116328	35
26	.783788	2.75	.899854	1.61	.883934	4.36	.116066	34
27	.783953	2.75	.899757	1.61	.884196	4.36	.115804	33
28	.784118	2.75	.899660	1.61	.884457	4.36	.115543	32
29	.784282	2.74	.899564	1.61	.884719	4.36	.115281	31
30	.784447	2.74	.899467	1.62	.884980	4.36	.115020	30
31	9.784612	2.74	9.899370	1.62	9.885242	4.36	10.114758	29
32	.784776	2.74	.899273	1.62	.885503	4.36	.114497	28
33	.784941	2.74	.899176	1.62	.885765	4.36	.114235	27
34	.785105	2.74	.899078	1.62	.886026	4.36	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.36	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.35	.113451	24
37	.785597	2.73	.898787	1.62	.886810	4.35	.113190	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.62	.887333	4.35	.112667	21
40	.786089	2.73	.898494	1.63	.887594	4.35	.112406	20
41	9.786252	2.72	9.898397	1.63	9.887855	4.35	10.112145	19
42	.786416	2.72	.898299	1.63	.888116	4.35	.111884	18
43	.786579	2.72	.898202	1.63	.888377	4.35	.111623	17
44	.786742	2.72	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889160	4.35	.110840	14
47	.787232	2.71	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.71	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.71	.897614	1.63	.889943	4.35	.110057	11
50	.787720	2.71	.897516	1.63	.890204	4.34	.109796	10
51	9.787883	2.71	9.897418	1.64	9.890465	4.34	10.109535	9
52	.788045	2.71	.897320	1.64	.890725	4.34	.109275	8
53	.788208	2.71	.897222	1 64	.890986	4.34	.109014	7
54	.788370	2.70	.897123	1 64	.891247	4.34	.108753	6
55	.788532	2.70	.897025	1.64	.891507	4.34	.108493	5
56	.788694	2.70	.896926	1.64	.891768	4.34	.108232	4
57	.788856	2.70	.896828	1.64	.892028	4.34	.107972	3
58	.789018	2.70	.896729	1 64	.892289	4.34	.107711	2
59	.789180	2.70	.896631	1.64	.892549	4.34	.107451	1
60	.789342	2.69	.896532	1.64	.892810	4.34	.107190	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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38°

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.789342	2.69	9.896532	1.64	9.892810	4.34	10.107190	60
1	.789504	2.69	.896433	1.65	.893070	4.34	.106930	59
2	.789665	2.69	.896335	1.65	.893331	4.34	.106669	58
3	.789827	2.69	.896236	1.65	.893591	4.34	.106409	57
4	.789988	2.69	.896137	1.65	.893851	4.34	.106149	56
5	.790149	2.69	.896038	1.65	.894111	4.34	.105889	55
6	.790310	2.68	.895939	1.65	.894371	4.34	.105629	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.68	.895741	1.65	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	.790954	2.68	.895542	1.65	.895412	4.33	.104588	50
11	9.791115	2.68	9.895443	1.66	9.895672	4.33	10.104328	49
12	.791275	2.67	.895343	1.66	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.66	.896192	4.33	.103808	47
14	.791596	2.67	.895145	1.66	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.66	.896712	4.33	.103288	45
16	.791917	2.67	.894945	1.66	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.66	.897231	4.33	.102769	43
18	.792237	2.66	.894746	1.66	.897491	4.33	.102509	42
19	.792397	2.66	.894646	1.66	.897751	4.33	.102249	41
20	.792557	2.66	.894546	1.66	.898010	4.33	.101990	40
21	9.792716	2.66	9.894446	1.67	9.898270	4.33	10.101730	39
22	.792876	2.66	.894346	1.67	.898530	4.33	.101470	38
23	.793035	2.66	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	.793354	2.65	.894046	1.67	.899308	4.32	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.67	.899827	4.32	.100173	33
28	.793832	2.65	.893745	1.67	.900086	4.32	.099914	32
29	.793991	2.65	.893645	1.67	.900346	4.32	.099654	31
30	.794150	2.64	.893544	1.67	.900605	4.32	.099395	30
31	9.794308	2.64	9.893444	1.68	9.900864	4.32	10.099136	29
32	.794467	2.64	.893343	1.68	.901124	4.32	.098876	28
33	.794626	2.64	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.64	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.64	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.64	.892940	1.68	.902160	4.32	.097840	24
37	.795259	2.63	.892839	1.68	.902419	4.32	.097581	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.68	.902938	4.32	.097062	21
40	.795733	2.63	.892536	1.68	.903197	4.31	.096803	20
41	9.795891	2.63	9.892435	1.69	9.903455	4.31	10.096545	19
42	.796049	2.63	.892334	1.69	.903714	4.31	.096286	18
43	.796206	2.63	.892233	1.69	.903973	4.31	.096027	17
44	.796364	2.62	.892132	1.69	.904232	4.31	.095768	16
45	.796521	2.62	.892030	1.69	.904491	4.31	.095509	15
46	.796679	2.62	.891929	1.69	.904750	4.31	.095250	14
47	.796836	2.62	.891827	1.69	.905008	4.31	.094992	13
48	.796993	2.62	.891726	1.69	.905267	4.31	.094733	12
49	.797150	2.61	.891624	1.69	.905526	4.31	.094474	11
50	.797307	2.61	.891523	1.70	.905784	4.31	.094216	10
51	9.797464	2.61	9.891421	1.70	9.906043	4.31	10.093957	9
52	.797621	2.61	.891319	1.70	.906302	4.31	.093698	8
53	.797777	2.61	.891217	1.70	.906560	4.31	.093440	7
54	.797934	2.61	.891115	1.70	.906819	4.31	.093181	6
55	.798091	2.61	.891013	1.70	.907077	4.31	.092923	5
56	.798247	2.61	.890911	1.70	.907336	4.31	.092664	4
57	.798403	2.60	.890809	1.70	.907594	4.31	.092406	3
58	.798560	2.60	.890707	1.70	.907852	4.31	.092148	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60	.798872	2.60	.890503	1.70	.908369	4.30	.091631	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

51°

M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.798872	2.60	9.890503	1.70	9.908369	4.30	10.091631	60
1	.799028	2.60	.890400	1.71	.908628	4.30	.091372	59
2	.799184	2.60	.890298	1.71	.908886	4.30	.091114	58
3	.799339	2.59	.890195	1.71	.909144	4.30	.090856	57
4	.799495	2.59	.890093	1.71	.909402	4.30	.090598	56
5	.799651	2.59	.889990	1.71	.909660	4.30	.090340	55
6	.799806	2.59	.889888	1.71	.909918	4.30	.090082	54
7	.799962	2.59	.889785	1.71	.910177	4.30	.089823	53
8	.800117	2.59	.889682	1.71	.910435	4.30	.089565	52
9	.800272	2.58	.889579	1.71	.910693	4.30	.089307	51
10	.800427	2.58	.889477	1.71	.910951	4.30	.089049	50
11	9.800582	2.58	9.889374	1.72	9.911209	4.30	10.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.72	.911724	4.30	.088276	47
14	.801047	2.58	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.57	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.72	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.29	.086986	42
19	.801819	2.57	.888548	1.72	.913271	4.29	.086729	41
20	.801973	2.57	.888444	1.73	.913529	4.29	.086471	40
21	9.802128	2.57	9.888341	1.73	9.913787	4.29	10.086213	39
22	.802282	2.56	.888237	1.73	.914044	4.29	.085956	38
23	.802436	2.56	.888134	1.73	.914302	4.29	.085698	37
24	.802589	2.56	.888030	1.73	.914560	4.29	.085440	36
25	.802743	2.56	.887926	1.73	.914817	4.29	.085183	35
26	.802897	2.56	.887822	1.73	.915075	4.29	.084925	34
27	.803050	2.56	.887718	1.73	.915332	4.29	.084668	33
28	.803204	2.56	.887614	1.73	.915590	4.29	.084410	32
29	.803357	2.55	.887510	1.73	.915847	4.29	.084153	31
30	.803511	2.55	.887406	1.74	.916104	4.29	.083896	30
31	9.803664	2.55	9.887302	1.74	9.916362	4.29	10.083638	29
32	.803817	2.55	.887198	1.74	.916619	4.29	.083381	28
33	.803970	2.55	.887093	1.74	.916877	4.29	.083123	27
34	.804123	2.55	.886989	1.74	.917134	4.29	.082866	26
35	.804276	2.54	.886885	1.74	.917391	4.29	.082609	25
36	.804428	2.54	.886780	1.74	.917648	4.29	.082352	24
37	.804581	2.54	.886676	1.74	.917905	4.29	.082095	23
38	.804734	2.54	.886571	1.74	.918163	4.28	.081837	22
39	.804886	2.54	.886466	1.74	.918420	4.28	.081580	21
40	.805039	2.54	.886362	1.75	.918677	4.28	.081323	20
41	9.805191	2.54	9.886257	1.75	9.918934	4.28	10.081066	19
42	.805343	2.53	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.53	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.75	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.75	.920990	4.28	.079010	11
50	.806557	2.52	.885311	1.76	.921247	4.28	.078753	10
51	9.806709	2.52	9.885205	1.76	9.921503	4.28	10.078497	9
52	.806860	2.52	.885100	1.76	.921760	4.28	.078240	8
53	.807011	2.52	.884994	1.76	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.76	.922274	4.28	.077726	6
55	.807314	2.52	.884783	1.76	.922530	4.28	.077470	5
56	.807465	2.51	.884677	1.76	.922787	4.28	.077213	4
57	.807615	2.51	.884572	1.76	.923044	4.28	.076956	3
58	.807766	2.51	.884466	1.76	.923300	4.28	.076700	2
59	.807917	2.51	.884360	1.76	.923557	4.27	.076443	1
60	.808067	2.51	.884254	1.77	.923813	4.27	.076187	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

M	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.808067	2.51	9.884254	1.77	9.923813	4.27	10.076187	60
1	.808218	2.51	.884148	1.77	.924070	4.27	.075930	59
2	.808368	2.51	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.77	.924583	4.27	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.77	.925352	4.27	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.77	.925865	4.27	.074135	52
9	.809419	2.49	.883297	1.78	.926122	4.27	.073878	51
10	.809569	2.49	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.49	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.49	.882977	1.78	.926890	4.27	.073110	48
13	.810017	2.49	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.49	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.79	.928427	4.27	.071573	42
19	.810912	2.48	.882229	1.79	.928683	4.27	.071317	41
20	.811061	2.48	.882121	1.79	.928940	4.27	.071060	40
21	9.811210	2.48	9.882014	1.79	9.929196	4.27	10.070804	39
22	.811358	2.47	.881907	1.79	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.79	.929708	4.27	.070292	37
24	.811655	2.47	.881692	1.79	.929964	4.26	.070036	36
25	.811804	2.47	.881584	1.79	.920220	4.26	.069780	35
26	.811952	2.47	.881477	1.79	.930475	4.26	.069525	34
27	.812100	2.47	.881369	1.79	.930731	4.26	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.26	.069013	32
29	.812396	2.46	.881153	1.80	.931243	4.26	.068757	31
30	.812544	2.46	.881046	1.80	.931499	4.26	.068501	30
31	9.812692	2.46	9.880938	1.80	9.931755	4.26	10.068245	29
32	.812840	2.46	.880830	1.80	.932010	4.26	.067990	28
33	.812988	2.46	.880722	1.80	.932266	4.26	.067734	27
34	.813135	2.46	.880613	1.80	.932522	4.26	.067478	26
35	.813283	2.46	.880505	1.80	.932778	4.26	.067222	25
36	.813430	2.45	.880397	1.80	.933033	4.26	.066967	24
37	.813578	2.45	.880289	1.81	.933289	4.26	.066711	23
38	.813725	2.45	.880180	1.81	.933545	4.26	.066455	22
39	.813872	2.45	.880072	1.81	.933800	4.26	.066200	21
40	.814019	2.45	.879963	1.81	.934056	4.26	.065944	20
41	9.814166	2.45	9.879855	1.81	9.934311	4.26	10.065689	19
42	.814313	2.45	.879746	1.81	.934567	4.26	.065433	18
43	.814460	2.44	.879637	1.81	.934823	4.26	.065177	17
44	.814607	2.44	.879529	1.81	.935078	4.26	.064922	16
45	.814753	2.44	.879420	1.81	.935333	4.26	.064667	15
46	.814900	2.44	.879311	1.81	.935589	4.26	.064411	14
47	.815046	2.44	.879202	1.82	.935844	4.26	.064156	13
48	.815193	2.44	.879093	1.82	.936100	4.26	.063900	12
49	.815339	2.44	.878984	1.82	.936355	4.26	.063645	11
50	.815485	2.43	.878875	1.82	.936610	4.26	.063390	10
51	9.815631	2.43	9.878766	1.82	9.936866	4.25	10.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.25	.062879	8
53	.815924	2.43	.878547	1.82	.937376	4.25	.062624	7
54	.816069	2.43	.878438	1.82	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.25	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.42	.877999	1.83	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	.816943	2.42	.877780	1.83	.939163	4.25	.060837	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.42	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.41	.877340	1.83	.940183	4.25	.059817	56
5	.817668	2.41	.877230	1.84	.940438	4.25	.059562	55
6	.817813	2.41	.877120	1.84	.940694	4.25	.059306	54
7	.817958	2.41	.877010	1.84	.940949	4.25	.059051	53
8	.818103	2.41	.876899	1.84	.941204	4.25	.058796	52
9	.818247	2.41	.876789	1.84	.941458	4.25	.058542	51
10	.818392	2.41	.876678	1.84	.941714	4.25	.058286	50
11	9.818536	2.40	9.876568	1.84	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.84	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.84	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.85	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.25	.056502	43
18	.819545	2.39	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.39	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.39	.875571	1.85	.944262	4.25	.055738	40
21	9.819976	2.39	9.875459	1.85	9.944517	4.25	10.055483	39
22	.820120	2.39	.875348	1.85	.944771	4.24	.055229	38
23	.820263	2.39	.875237	1.85	.945026	4.24	.054974	37
24	.820406	2.39	.875126	1.86	.945281	4.24	.054719	36
25	.820550	2.38	.875014	1.86	.945535	4.24	.054465	35
26	.820693	2.38	.874903	1.86	.945790	4.24	.054210	34
27	.820836	2.38	.874791	1.86	.946045	4.24	.053955	33
28	.820979	2.38	.874680	1.86	.946299	4.24	.053701	32
29	.821122	2.38	.874568	1.86	.946554	4.24	.053446	31
30	.821265	2.38	.874456	1.86	.946808	4.24	.053192	30
31	9.821407	2.38	9.874344	1.86	9.947063	4.24	10.052937	29
32	.821550	2.38	.874232	1.87	.947318	4.24	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.24	.052428	27
34	.821835	2.37	.874009	1.87	.947826	4.24	.052174	26
35	.821977	2.37	.873896	1.87	.948081	4.24	.051919	25
36	.822120	2.37	.873784	1.87	.948336	4.24	.051664	24
37	.822262	2.37	.873672	1.87	.948590	4.24	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.24	.051156	22
39	.822546	2.37	.873448	1.87	.949099	4.24	.050901	21
40	.822688	2.36	.873335	1.87	.949353	4.24	.050647	20
41	9.822830	2.36	9.873223	1.87	9.949607	4.24	10.050393	19
42	.822972	2.36	.873110	1.88	.949862	4.24	.050138	18
43	.823114	2.36	.872998	1.88	.950116	4.24	.049884	17
44	.823255	2.36	.872885	1.88	.950370	4.24	.049630	16
45	.823397	2.36	.872772	1.88	.950625	4.24	.049375	15
46	.823539	2.36	.872659	1.88	.950879	4.24	.049121	14
47	.823680	2.35	.872547	1.88	.951133	4.24	.048867	13
48	.823821	2.35	.872434	1.88	.951388	4.24	.048612	12
49	.823963	2.35	.872321	1 88	.951642	4.24	.048358	11
50	.824104	2.35	.872208	1.88	.951896	4.24	.048104	10
51	9.824245	2.35	9.872095	1.89	9.952150	4.24	10.047850	9
52	.824386	2.35	.871981	1.89	.952405	4.24	.047595	8
53	.824527	2.35	.871868	1 89	.952659	4.24	.047341	7
54	.824668	2.34	.871755	1.89	.952913	4.24	.047087	6
55	.824808	2.34	.871641	1.89	.953167	4.23	.046833	5
56	.824949	2.34	.871528	1.89	.953421	4.23	.046579	4
57	.825090	2.34	.871414	1.89	.953675	4.23	.046325	3
58	.825230	2.34	.871301	1.89	.953929	4.23	.046071	2
59	.825371	2.34	871187	1.89	.954183	4.23	.045817	1
60	.825511	2.34	.871073	1.90	.954437	4.23	.045563	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	10.045563	60
1	.825651	2.33	.870960	1.90	.954691	4.23	.045309	59
2	.825791	2.33	.870846	1.90	.954945	4.23	.045055	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955707	4.23	.044293	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.90	.956215	4.23	.043785	53
8	.826631	2.33	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.32	.870047	1.91	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.91	.956977	4.23	.043023	50
11	9.827049	2.32	9.869818	1.91	9.957231	4.23	10.042769	49
12	.827189	2.32	.869704	1.91	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.91	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.91	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.91	.958246	4.23	.041754	45
16	.827745	2.32	.869245	1.91	.958500	4.23	.041500	44
17	.827884	2.31	.869130	1.91	.958754	4.23	.041246	43
18	.828023	2.31	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.31	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.31	.868785	1.92	.959516	4.23	.040484	40
21	9.828439	2.31	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.31	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.31	.868440	1.92	.960277	4.23	.039723	37
24	.828855	2.30	.868324	1.92	.960531	4.23	.039469	36
25	.828993	2.30	.868209	1.92	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961291	4.23	.038709	33
28	.829407	2.30	.867862	1.93	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.23	.038201	31
30	.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
31	9.829821	2.29	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.29	.867399	1.93	.962560	4.23	.037440	28
33	.830097	2.29	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.29	.867167	1.93	.963067	4.23	.036933	26
35	.830372	2.29	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.29	.866935	1.94	.963574	4.23	.036426	24
37	.830646	2.29	.866819	1.94	.963827	4.23	.036173	23
38	.830784	2.29	.866703	1.94	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.94	.964335	4.23	.035665	21
40	.831058	2.28	.866470	1.94	.964588	4.22	.035412	20
41	9.831195	2.28	9.866353	1.94	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.94	.965095	4.22	.034905	18
43	.831469	2.28	.866120	1.94	.965349	4.22	.034651	17
44	.831606	2.28	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.22	.034145	15
46	.831879	2.28	.865770	1.95	.966105	4.22	.033891	14
47	.832015	2.27	.865653	1.95	.966362	4.22	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.27	.865419	1.95	.966869	4.22	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9.832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.95	.967629	4.22	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.26	.864833	1.96	.968136	4.22	.031864	6
55	.833105	2.26	.864716	1.96	.968389	4.22	.031611	5
56	.833241	2.26	.864598	1.96	.968643	4.22	.031357	4
57	.833377	2.26	.864481	1.96	.968896	4.22	.031104	3
58	.833512	2.26	.864363	1.96	.969149	4.22	.030851	2
59	.833648	2.26	.864245	1.96	.969403	4.22	.030597	1
60	.833783	2.26	.864127	1.96	.969656	4.22	.030344	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M.

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M.	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	10.030344	60
1	.833919	2.25	.864010	1.96	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.22	.029838	58
3	.834189	2.25	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	029331	56
5	.834460	2.25	.863538	1.97	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.22	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.25	.863183	1.97	.971682	4.22	.028318	52
9	.834999	2.24	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.24	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.24	9.862827	1.98	9.972441	4.22	10.027559	49
12	.835403	2.24	.862709	1.98	.972694	4.22	.027306	48
13	.835538	2.24	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.24	.862471	1.98	.973201	4.22	.026799	46
15	.835807	2.24	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.24	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.22	.025534	41
20	.836477	2.23	.861758	1.99	.974719	4.22	.025281	40
21	9.836611	2.23	9.861638	1.99	9.974973	4.22	10.025027	39
22	.836745	2.23	.861519	1 99	.975226	4.22	.024774	38
23	.836878	2.23	.861400	1.99	.975479	4.22	.024521	37
24	.837012	2.22	.861280	1.99	.975732	4.22	.024268	36
25	.837146	2.22	.861161	1.99	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.99	.976238	4.22	.023762	34
27	.837412	2.22	.860922	1.99	.976491	4.22	.023509	33
28	.837546	2.22	.860802	1.99	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.21	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.21	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.21	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.21	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.21	.859842	2.00	.978768	4.22	.021232	24
37	.838742	2.21	.859721	2.01	.979021	4.22	.020979	23
38	.838875	2.21	.859601	2.01	.979274	4.22	.020726	22
39	.839007	2.21	.859480	2.01	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2 01	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2 01	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.01	.980286	4.22	.019714	18
43	.839536	2.20	.858998	2.01	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.01	.980791	4.21	.019209	16
45	.839800	2.20	.858756	2 02	.981044	4.21	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.21	.018703	14
47	.840064	2.19	.858514	2.02	.981550	4.21	.018450	13
48	.840196	2.19	.858393	2 02	.981803	4.21	.018197	12
49	.840328	2.19	.858272	2.02	.982056	4.21	.017944	11
50	.840459	2.19	.858151	2 02	.982309	4.21	.017691	10
51	9.840591	2.19	9.858029	2 02	9.982562	4.21	10.017438	9
52	.840722	2.19	.857908	2.02	.982814	4.21	.017186	8
53	.840854	2.19	.857786	2.02	.983067	4.21	.016933	7
54	.840985	2.19	.857665	2.03	.983320	4.21	.016680	6
55	.841116	2.18	.857543	2.03	.983573	4.21	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.21	.016174	4
57	.841378	2.18	.857300	2 03	.984079	4.21	.015921	3
58	.841509	2.18	.857178	2.03	.984331	4.21	.015669	2
59	.841640	2.18	.857056	2.03	.984584	4.21	.015416	1
60	.841771	2.18	.856934	2 03	.984837	4.21	.015163	0
	Cosine	D.	Sine.	D.	Cotang.	D.	Tang.	M.

46°

M	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.21	.014910	59
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657	58
3	.842163	2.17	.856568	2.04	.985596	4.21	.014404	57
4	.842294	2.17	.856446	2.04	.985848	4.21	.014152	56
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899	55
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646	54
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393	53
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140	52
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.21	.012635	50
11	9.843206	2.16	9.855588	2.05	9.987618	4.21	10.012382	49
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129	48
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877	47
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624	46
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371	45
16	.843855	2.16	.854973	2.05	.988882	4.21	.011118	44
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866	43
18	.844114	2.15	.854727	2.06	.989387	4.21	.010613	42
19	.844243	2.15	.854603	2.06	.989640	4.21	.010360	41
20	.844372	2.15	.854480	2.06	.989893	4.21	.010107	40
21	9.844502	2.15	9.854356	2.06	9.990145	4.21	10.009855	39
22	.844631	2.15	.854233	2.06	.990398	4.21	.009602	38
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349	37
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097	36
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844	35
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591	34
27	.845276	2.14	.853614	2.07	.991662	4.21	.008338	33
28	.845405	2.14	.853490	2.07	.991914	4.21	.008086	32
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833	31
30	.845662	2.14	.853242	2.07	.992420	4.21	.007580	30
31	9.845790	2.14	9.853118	2.07	9.992672	4.21	10.007328	29
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075	28
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822	27
34	.846175	2.14	.852745	2.07	.993430	4.21	.006570	26
35	.846304	2.14	.852620	2.07	.993683	4.21	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.21	.006064	24
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306	21
40	.846944	2.13	.851997	2.08	.994947	4.21	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.21	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548	18
43	.847327	2.13	.851622	2.08	.995705	4.21	.004295	17
44	.847454	2.12	.851497	2.09	.995957	4.21	.004043	16
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790	15
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537	14
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285	13
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032	12
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779	11
50	.848218	2.12	.850745	2.09	.997473	4.21	.002527	10
51	9.848345	2.12	9.850619	2.09	9.997726	4.21	10.002274	9
52	.848472	2.11	.850493	2.10	.997979	4.21	.002021	8
53	.848599	2.11	.850368	2.10	.998231	4.21	.001769	7
54	.848726	2.11	.850242	2.10	.998484	4.21	.001516	6
55	.848852	2.11	.850116	2.10	.998737	4.21	.001263	5
56	.848979	2.11	.849990	2.10	.998989	4.21	.001011	4
57	.849106	2.11	.849864	2.10	.999242	4.21	.000758	3
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505	2
59	.849359	2.11	.849611	2.10	.999748	4.21	.000253	1
60	.849485	2.11	.849485	2.10	10.000000	4.21	10.000000	0
	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	M

45°

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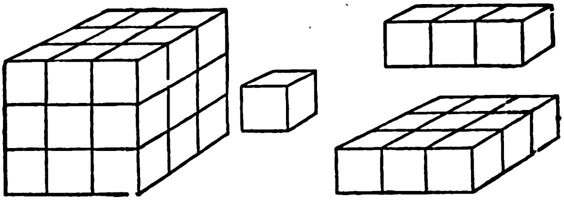

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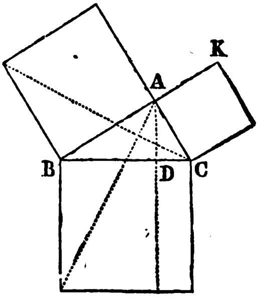

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